LAAS CNRS

Resource Constraints in Scheduling

Emmanuel Hebrard

Content: constraint programming for scheduling

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Part I: Propagation of resource constraints

- Constraint programming view of scheduling
- Main bibliographic sources
  - Constraint-based Scheduling (Baptiste, Le Pape, and Nuijten, 2001)
  - Petr Vilím's thesis (Vilím, 2007)

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Part III: A glimpse of other types of resources

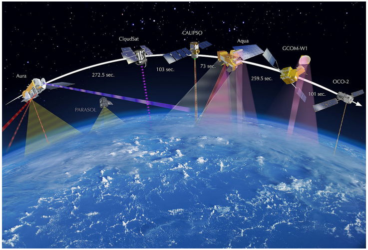
- Resources come in every form and shape, Rosetta/Philae example

What is scheduling?

- "Allocating scarce resources to activities over time" (Baker, 1974)

File download from observation satellites

- Jobs: Files to transfer
- Resources:
  - Download channels: at most that many simultaneous downloads
  - Memory banks: cannot download two files stored on the same memory bank simultaneously
- Download as much data as possible within a given time window



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- Jobs: Scientific experiments
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  - Batteries: threshold on the instant energy consumption
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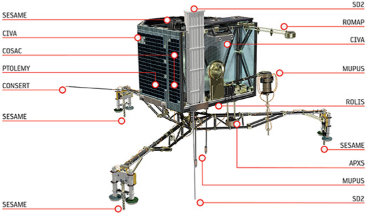

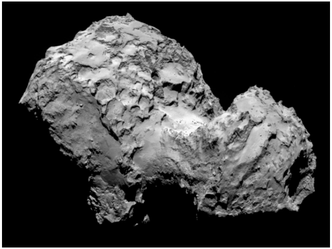

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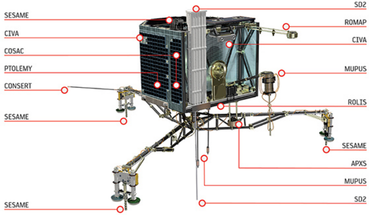
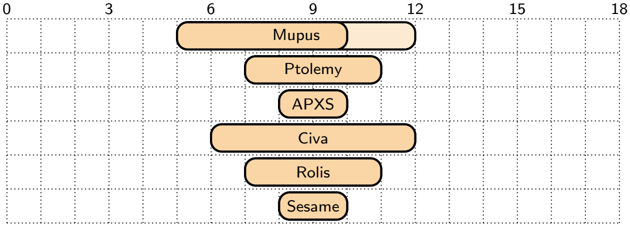


Notations

- A *task* i is represented as a rectangle
  - width represents processing time p_i (possibly variable)



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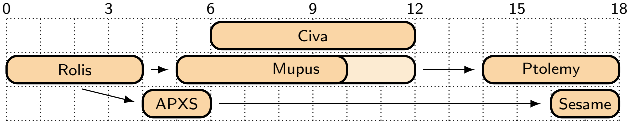

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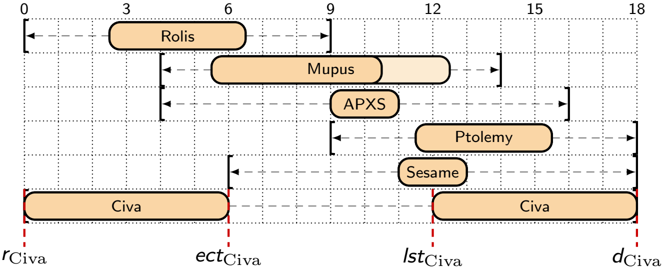

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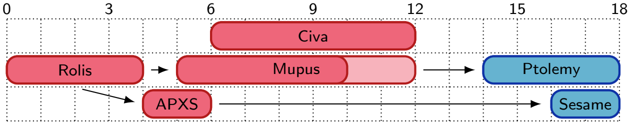

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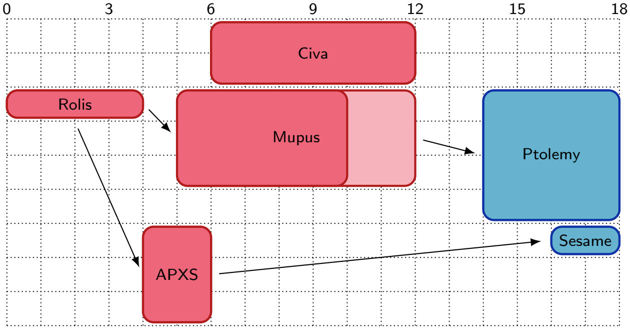

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- Within Constraint Programming: not so important
  - Handled by a constraint
  - Depend on start and end times of tasks

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Constraint vs Algorithm

- Constraints & models written using *variables* symbols (s_i, e_i)
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- Notion of consistency: constraints are sufficient to define the result of propagation

Bound consistency

Constraint C over variables $\mathcal{X}$

- C: predicate defining a relation in $\mathbb{N}^{|\mathcal{X}|}$

Ex: $i \prec j$

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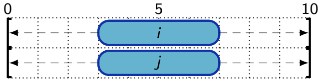

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- $\sigma : \mathcal{X} \mapsto \mathbb{N}$ with $\sigma(x) = t$
- valid $\iff \forall x \in \mathcal{X}\ \min(x) \leq \sigma(x) \leq \max(x)$
- consistent $\iff C(\sigma(\mathcal{X}))$

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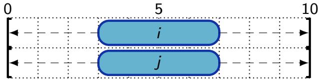

- Ex.: no valid and consistent bound support for $e_i = 7$
  - consistent: $\langle e_i : 7, s_j : 7\rangle$
  - valid: $\langle e_i : 7, s_j : 6\rangle$

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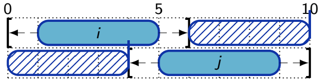

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- Result of propagation algorithm is entailed by "*bound consistency on* $e_i \leq s_j$"
- Its complexity is not

Resource Constraint

- Very rich taxonomy of resources
- Focus on Renewable, discrete, non-interruptible, cumulative resource

A model of CumulativeResource

- Processing time: require the resources for at least p_i time $\qquad s_i + p_i \leq e_i \; \forall i$
- Non preemption: cannot be interrupted $\qquad s_i + p_i \geq e_i \; \forall i$
- Bounds: release and due dates $\qquad r_i \leq s_i \leq e_i \leq d_i \; \forall i$
- Resource capacity: additive, upper bounded resource usage $\qquad \sum_{s_i \leq t \leq e_i} c_i \leq C \; \forall t$

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- **Philae**; battery power threshold: cumulative resource ($c_i \geq 1, C > 1$)

Coping with NP-hardness

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- The problem CUMULATIVERESOURCE is strongly NP-hard
  - So is bound consistency, since a bound support is a valid schedule

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A decomposition of CUMULATIVERESOURCE

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- Decomposition:
  - Enforcing bound consistency on CUMULATIVERESOURCE is NP-hard (global support)
  - Enforcing bound consistency on the model above is polynomial (local supports)

Time-tabling

- Notion of "compulsory part" (Lahrichi, 1982)
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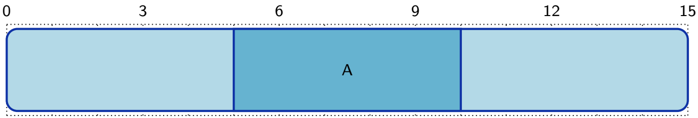
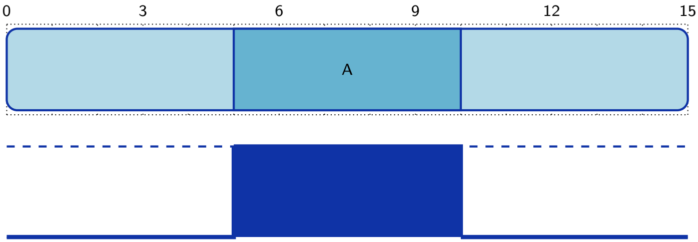


Time-tabling

- Notion of “compulsory part” (Lahrichi, 1982)
  - Period in which the task must be in process
- Notion of “time-tables” (Le Pape, 1988), “resource profile” (Fox, 1990), “resource histogram” (Caseau and Laburthe, 1996)
  - Minimum usage of the resource over time



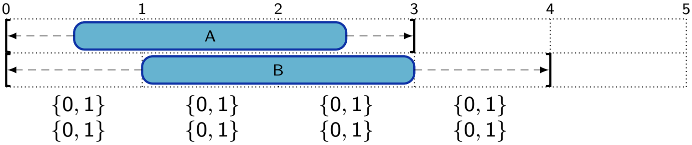
Time-tabling decomposition

- Boolean variables a_i^t standing for: "task i is in process at time t"
- Enforce bounds consistency on:

$$\forall i \quad s_i + p_i = e_i \quad \text{processing time \& non-preemption} \quad (1)$$

$$\forall t \quad \sum_{i \in \mathcal{T}} c_i a_i^t \leq C \quad \text{resource capacity} \quad (2)$$

$$\forall i \forall t \quad a_i^t \iff s_i \leq t \wedge t < e_i \quad \text{channeling} \quad (3)$$



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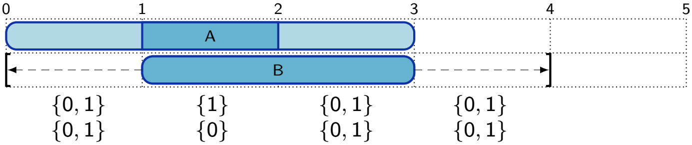

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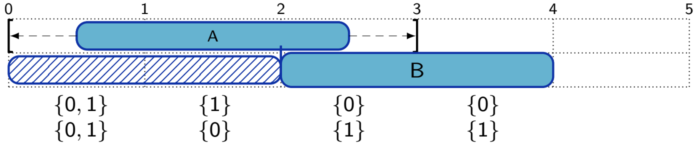

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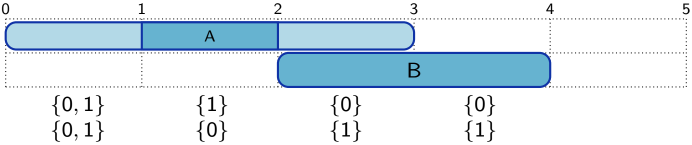

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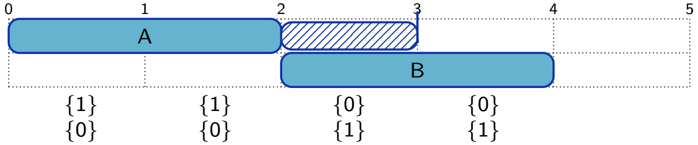

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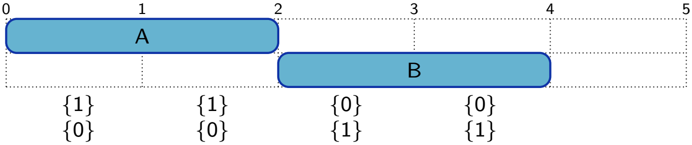

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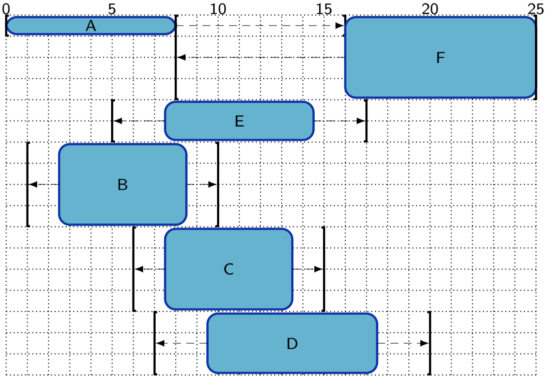
Time-tabling algorithm: satisfiability check

Resource profile



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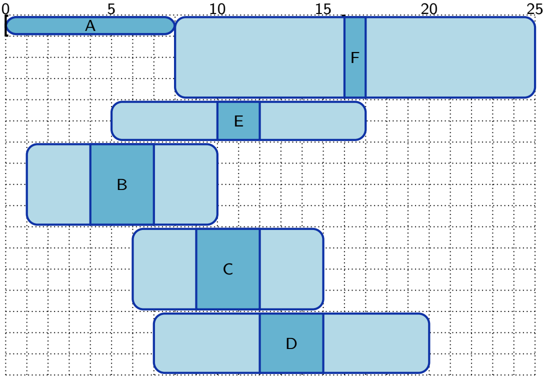

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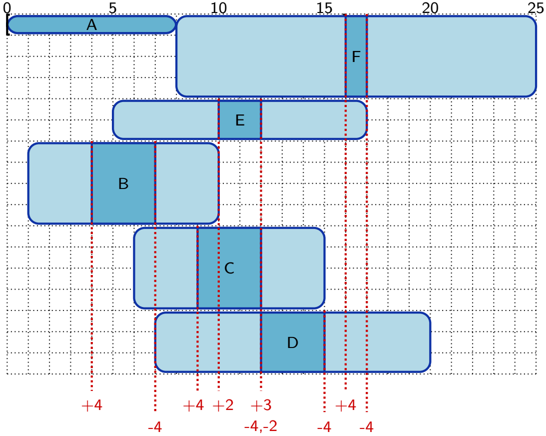

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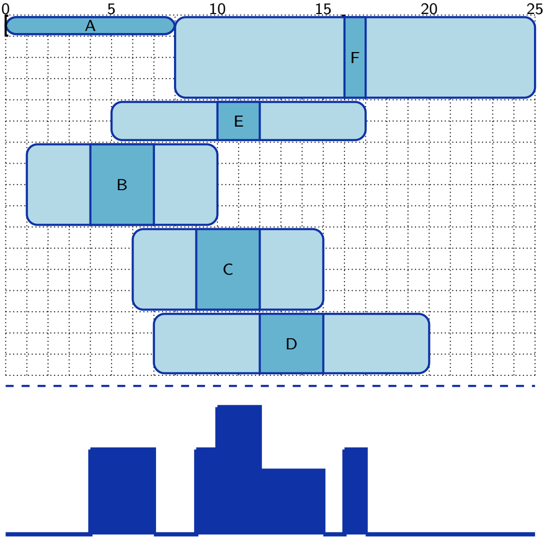

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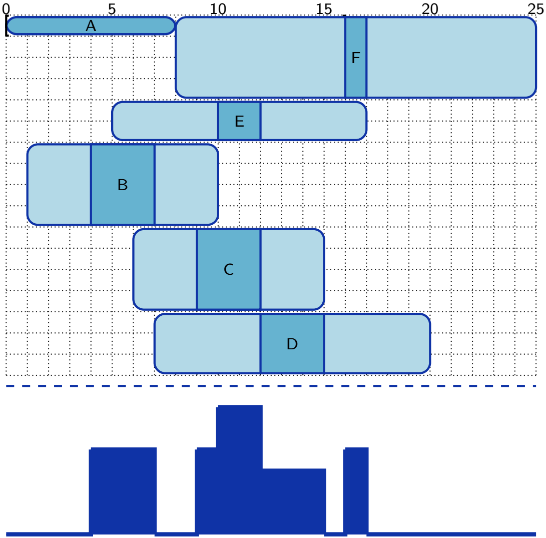
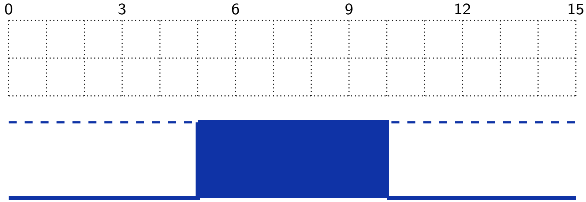
- Sweep algorithm (Beldiceanu and Carlsson, 2001) (more general)



Time-tabling algorithm: bound propagator

- Assume $C = 3$ and consider the profile interval $[5, 10) : 2$



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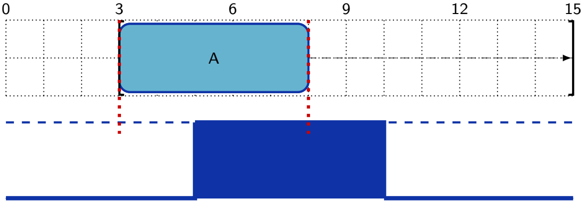

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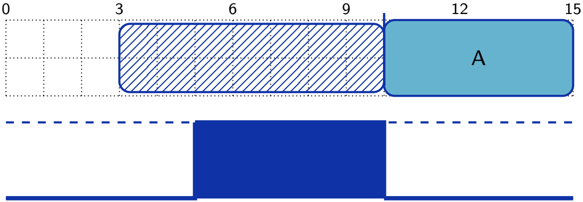

Time-tabling algorithm: bound propagator

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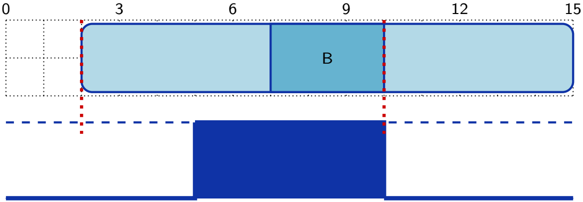
- A task B counted in the profile



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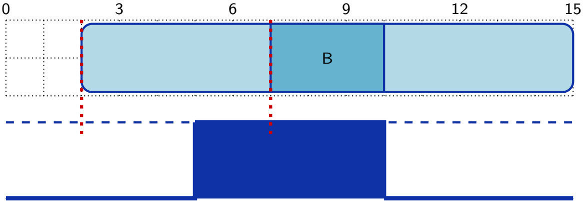
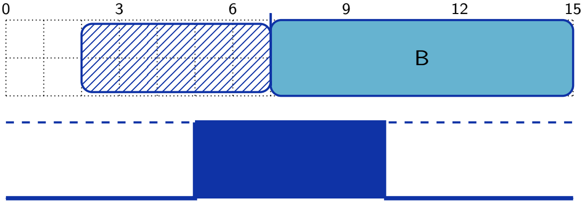


Time-tabling algorithm: bound propagator

- Assume $C = 3$ and consider the profile interval $[5, 10) : 2$
- A task B counted in the profile
- $[a, b)$ overlaps with $[r_i, \min(lst_i, ect_i)) \implies r_i = \min(b, lst_i)$



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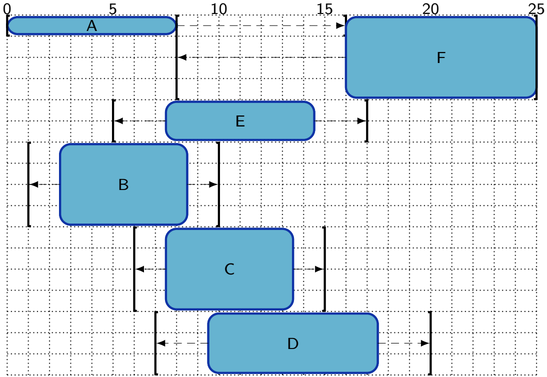

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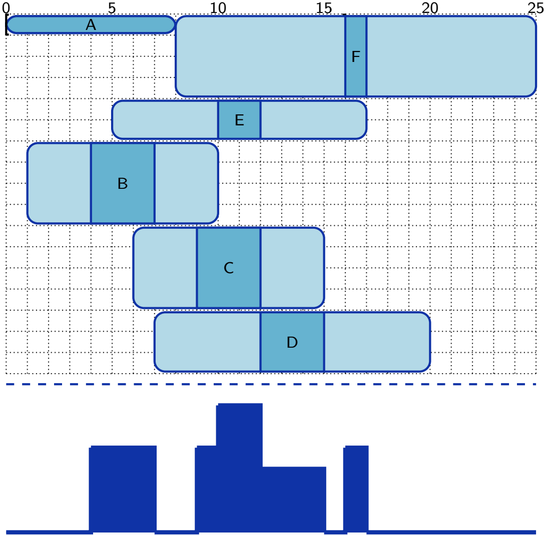

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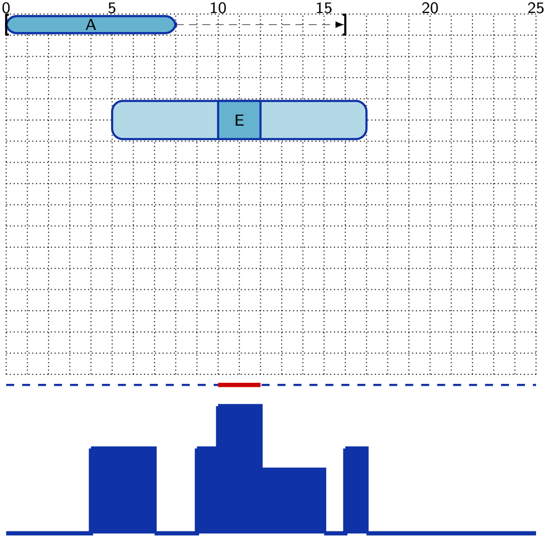

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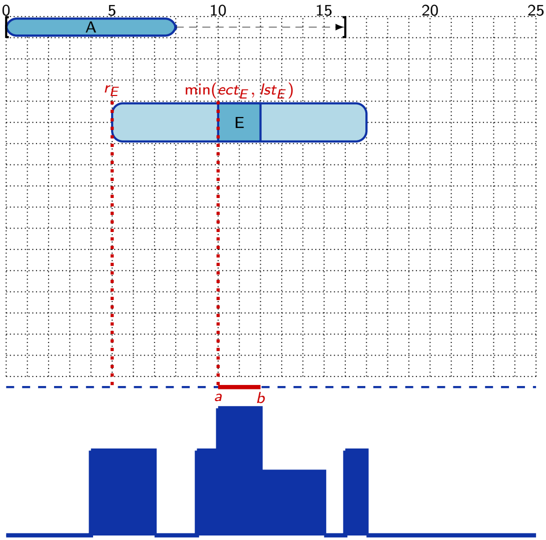

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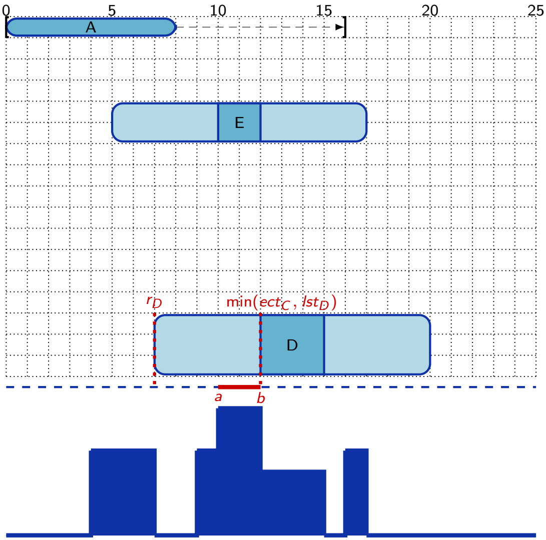

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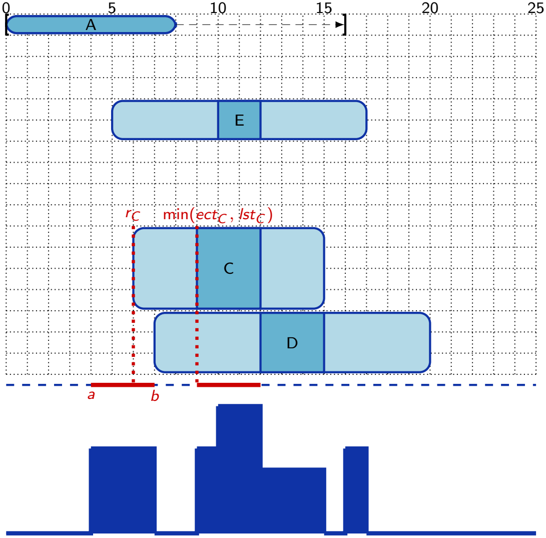

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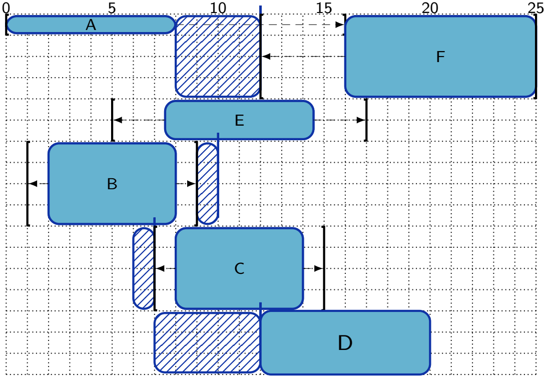

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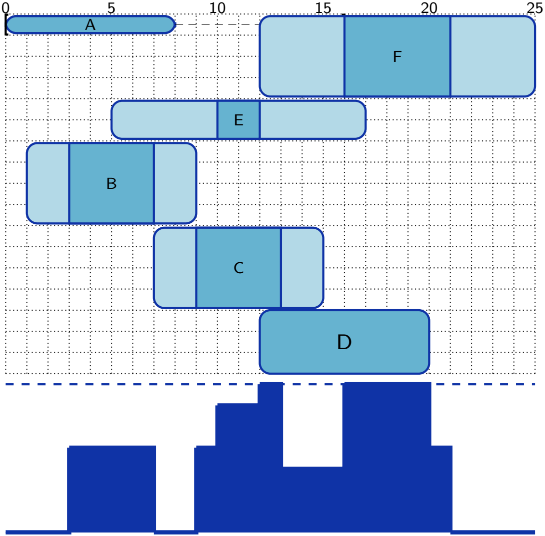

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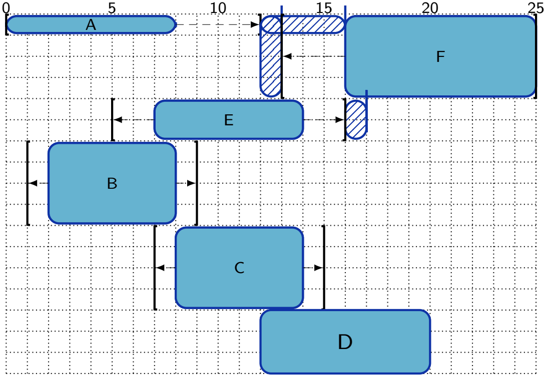

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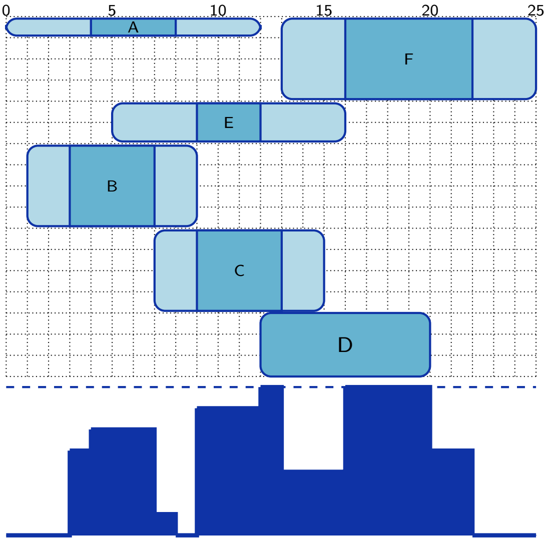

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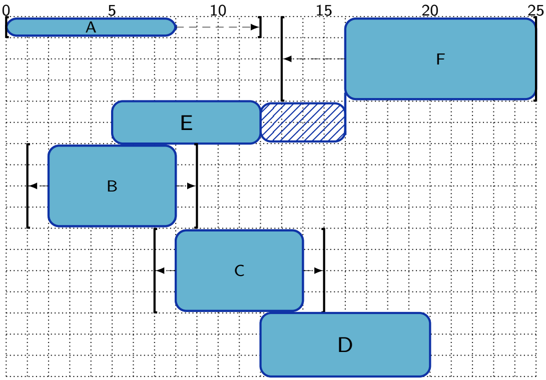

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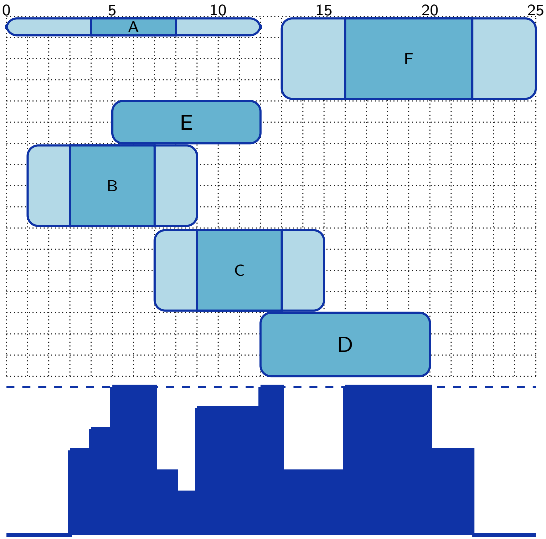


Time-tabling Algorithms

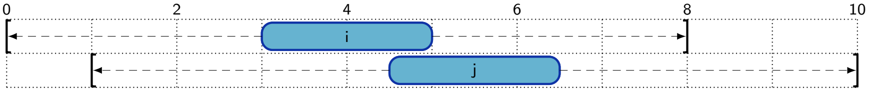
- Sweep algorithm (Beldiceanu and Carlsson, 2001)
- $O(n^2)$ synchornized sweep algoritm (**LetortEtAl12**)
- $O(n \log n)$ algorithm (Ouellet and Quimper, 2013)
- $O(n)$ algorithm (not pracical) and $O(n^2)$ efficient and simple algorithm (Gay, Hartert, and Schaus, 2015)

PySched – Available on GitHub

```python
outfile = open('tex/ex/timetabling.tex', 'w')

s = Schedule()

A = Task(s,duration=8,release=0,duedate=26,demand=1,label='A')
B = Task(s,duration=6,release=1,duedate=10,demand=4,label='B')
C = Task(s,duration=6,release=6,duedate=15,demand=4,label='C')
D = Task(s,duration=8,release=7,duedate=20,demand=3,label='D')
E = Task(s,duration=7,release=5,duedate=17,demand=2,label='E')
F = Task(s,duration=9,release=5,duedate=25,demand=4,label='F')

res = Resource(s, 'A', [A,B,C,D,E,F], capacity=7)

A << F

tt = Timetabling(res)

while True:
    s.save()
    if not tt.propagate():
        break

    s.latex(outfile, animated=True, precedences=False, pruning=True, offset=0, rows=[
    s.latex(outfile, animated=True, mandatory=True, profile=[res], precedences=False,
```

Disjunctive constraints

- Change the viewpoint (variables) from start times to precedences
- Notion of disjunctive graph (Roy and Sussman, 1964) central to (Carlier and Pinson, 1989)'s method
  - If *i* and *j* require the same exclusive resource



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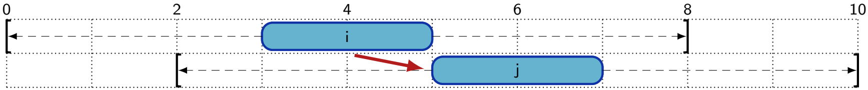

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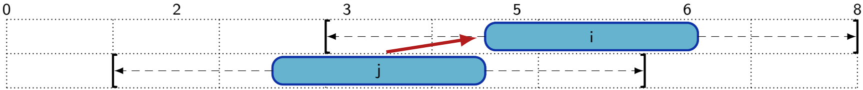

Disjunctive decomposition (single-machine)

$$\forall i < j \in \mathcal{T}, \quad b_{ij} \iff e_i \leq s_j$$
$$b_{ij} \neq b_{ji}$$

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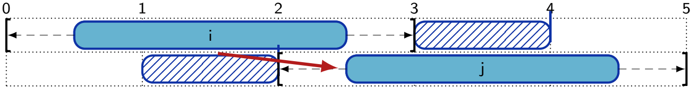

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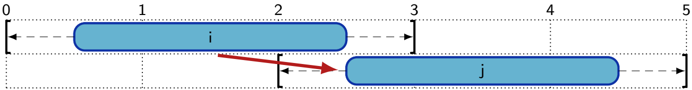

- Completeness of disjunctive propagation: deciding only b_{ij} variables is sufficient

Disjunctive algorithm

$$
\begin{aligned}
\forall i < j \in \mathcal{T}, \qquad [b_{ij}] &\implies i \prec j && \text{(post } i \prec j\text{'s propagator)}\\
\neg[b_{ij}] &\implies j \prec i && \text{(post } j \prec i\text{'s propagator)}\\
ect_j > lst_i &\implies [b_{ij}] &&\\
ect_i > lst_j &\implies \neg[b_{ij}] &&
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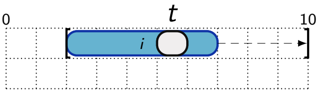
Strictly stronger than Time-tabling

- Suppose $a_i^t = 0$ and $e_i > t$



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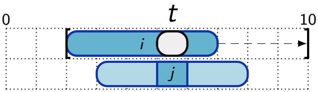
- Suppose $a_i^t = 0$ and $e_i > t$
  - $\exists j$ s.t. $a_j^t = 1$



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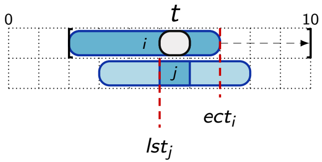
- Suppose $a_i^t = 0$ and $e_i > t$
  - $\exists j$ s.t. $a_j^t = 1$ hence $lst_j \leq t < ect_j$
  - Therefore, $lst_j < ect_i$ so b_{ij} must be false



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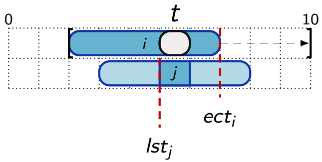
  - and thus $j \prec i$ is posted



Generalisation to cumulative resources

- Either $i \prec j$, $j \prec i$ or $i \not\prec j \wedge j \not\prec i$
  - Easy change: $b_{ii} \neq b_{ji}$ becomes $\neg(b_{ij} \wedge b_{ji})$, but not complete anymore!

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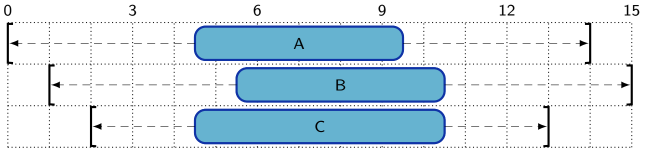
  - Easy change: $b_{ii} \neq b_{ji}$ becomes $\neg(b_{ij} \wedge b_{ji})$, but not complete anymore!
- To keep the property that start times do not need to be set:
  - For every minimal subset S of tasks such that $\sum_{i \in S} c_i > C$, post:

$$\bigvee_{i \neq j \in S} b_{ij}$$

  - Not used in practice

Energetic reasoning

- A lot of different forms and flavours
  - Preemptive relaxation
  - Fully Elastic relaxation
  - Partialy Elastic relaxation
  - Edge finding
  - Energetic reasoning



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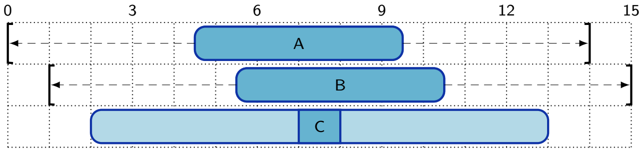

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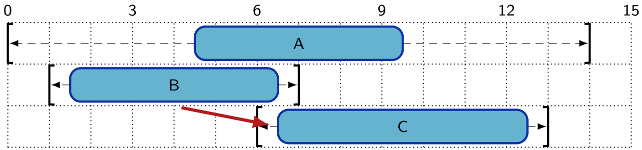

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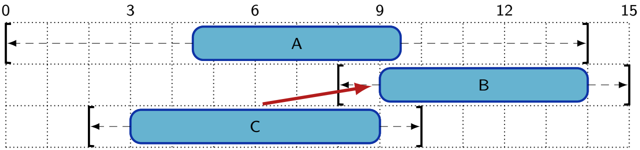

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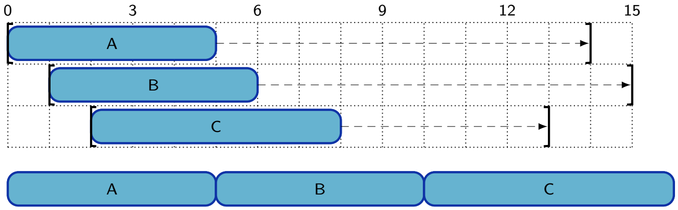

Energetic reasoning

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  - Preemptive relaxation
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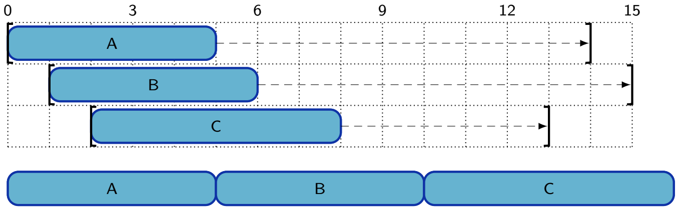
- Basic idea: view a (set of) task(s) as fluid quantity (energy)



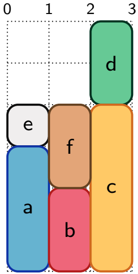
Preemptive relaxation (Federgruen and Groenevelt, 1986)

- Tasks can be interrupted (cut in vertical slices)

$$\begin{array}{rcl} \forall i \forall t \notin [r_i, d_i) & a_i^t = 0 & \text{bounds} \\ \forall t & \sum_i c_i a_i^t \leq C & \text{resource capacity} \\ \forall i & \sum_t a_i^t = p_i & \text{processing times} \end{array}$$

Properties of the preemptive relaxation

- NP-Hard, can encode BinPacking even with unit processing times
  - Capacity = number of bins
  - Consumption = item size
- Easy when $\forall i \; c_i = 1$: Maximum flow formulation
  - Gcc when $\forall i, \; p_i = 1$
  - AllDifferent when $\forall i, \; p_i = 1$ and $C = 1$



Fully elastic relaxation (Baptiste, Le Pape, and Nuijten, 1998)

- Tasks can be interrupted and resource usage is given by a total energy
- Replace Boolean "in process" a_i^t variables by integer "usage" u_i^t variables in $[0, c_i]$

$$
\begin{array}{rcl}
\forall i \forall t \notin [r_i, d_i) & u_i^t = 0 & \text{bounds} \\
\forall t & \sum_i u_i^t \leq C & \text{resource capacity} \\
\forall i & \sum_t u_i^t = p_i c_i & \text{energy}
\end{array}
$$

Properties of the fully elastic relaxation

- Polynomial
  - When $C = 1$, equivalent to preemptive relaxation and solved by Jackson Preemptive Schedule (Jackson, 1955) in $O(n \log n)$
  - When $C > 1$: equivalent reformulation to the $C = 1$ case
    - The horizon / time windows are multiplied by C
    - The processing time p_i is multiplied c_i

Partially elastic relaxation (Baptiste, Le Pape, and Nuijten, 1998)

- Strengthen the relaxation: constraint on energy of nested intervals

$\forall i \forall t \notin [r_i, d_i)$	$u_i^t = 0$	bounds
$\forall t$	$\sum_i u_i^t \leq C$	resource capacity
$\forall i$	$\sum_t u_i^t = p_i c_i$	energy
$\forall i \forall t \in [r_i, d_i)$	$\sum_{x<t} u_i^t \leq c_i(t - r_i)$	
$\forall i \forall t \in [r_i, d_i)$	$\sum_{x \geq t} u_i^t \leq c_i(d_i - t)$	

Properties of the partially elastic relaxation

- Equivalent to the SubsetSum bound (**Perregaard95**)
- Equivalent to the preemptive energetic reasoning (**LopezEtAl92**; Lopez, 1991)
- Algorithm in $O(n^2 \log n)$ (Baptiste, 1998)

Overload Checking decomposition

- The relaxation gives a satisfiability test but not a propagator
- Decomposition (Carlier, 1982):

$$\forall \Omega \subseteq \mathcal{T} \quad s_\Omega = \min(\{s_j \mid j \in \Omega\})$$
$$\forall \Omega \subseteq \mathcal{T} \quad e_\Omega = \max(\{e_j \mid j \in \Omega\})$$
$$\forall \Omega \subseteq \mathcal{T} \quad w_\Omega \leq C(e_\Omega - s_\Omega)$$

with $w_\Omega = \sum_{j \in \Omega} p_j c_j$

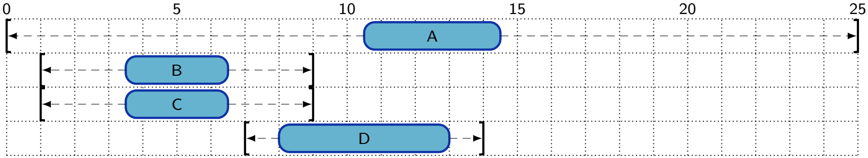
Theorem

Bound consistency on this decomposition fails
if and only if
the fully elastic decomposition is unsatisfiable

Edge Finding decomposition (single-machine)

- Channel to precedence variables (Carlier and Pinson, 1989), (Applegate and Cook, 1991)
  - adding i to Ω not last leads to overload $\implies$ i is last

$$\forall \Omega \subseteq \mathcal{T} \quad s_\Omega = min(\{s_j \mid j \in \Omega\})$$
$$\forall \Omega \subseteq \mathcal{T} \quad e_\Omega = max(\{e_j \mid j \in \Omega\})$$
$$\forall \Omega \subseteq \mathcal{T} \quad s_\Omega + p_\Omega \leq e_\Omega$$
$$\forall \Omega \subseteq \mathcal{T},\ \forall i \notin \Omega \quad b_{\Omega i} \iff \bigwedge_{j \in \Omega} e_j \leq s_i$$
$$\forall \Omega \subseteq \mathcal{T},\ \forall i \notin \Omega \quad s_{\Omega \cup \{i\}} + p_i + p_\Omega > e_\Omega \implies b_{\Omega i}$$



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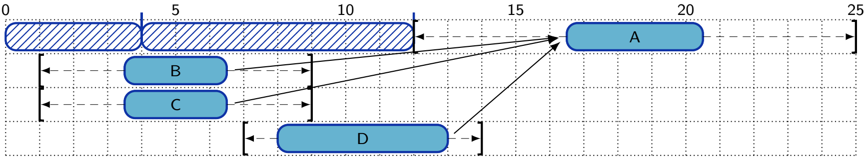

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$$
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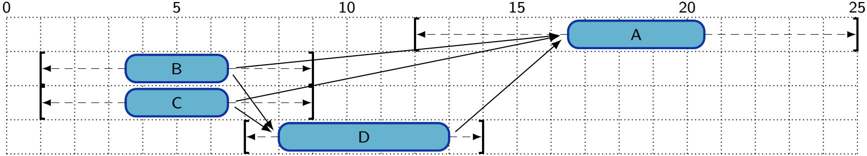
$$



Not first / Not last decomposition (single-machine)

- Reverse implication (Pinson, 1988)
  - adding i to Ω last leads to overload $\implies$ i is not last

$$\forall \Omega \subseteq \mathcal{T} \qquad s_\Omega = min(\{s_j \mid j \in \Omega\})$$

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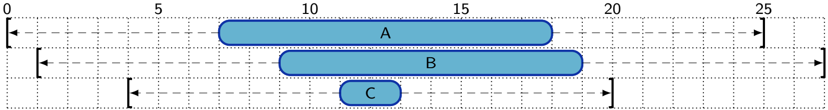

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$$\forall \Omega \subseteq \mathcal{T} \qquad s_\Omega = mjn(\{s_j \mid j \in \Omega\})$$

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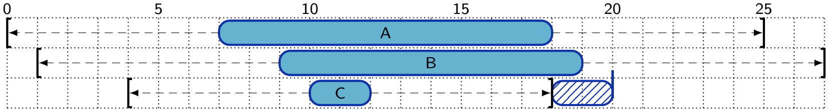

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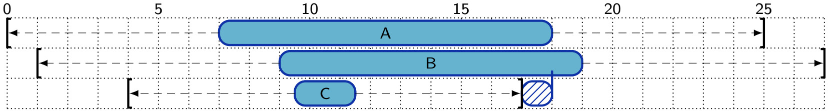


Overload Checking algorithm (Vilím, Barták, and Čepek, 2004)

- Definitions

$$\begin{aligned} ect_\Omega &= \max\{r_{\Omega'} + p_{\Omega'} \mid \Omega' \subseteq \Omega\} \\ \mathcal{T}|_j &= \{i \in \mathcal{T} \mid d_i \leq d_j\} \end{aligned}$$

- Reformulation

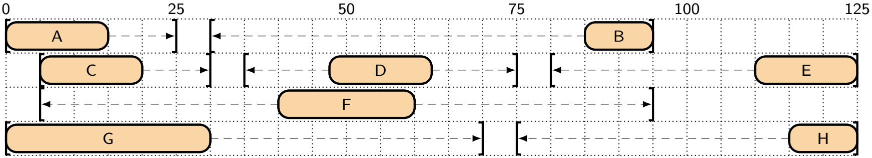
$$\forall \Omega \subseteq \mathcal{T}\ s_\Omega + p_\Omega \leq e_\Omega \iff \forall j \in \mathcal{T}\ ect_{\mathcal{T}|_j} \leq d_j$$

- May not seem like a big progress
  - Check $2^{|\mathcal{T}|}$ relations
  - Check $|\mathcal{T}|$ relations, each requiring to compute the max of $2^{|\mathcal{T}|}$ elements

Overload Checking – Dynamic Programming

$$ect_\Omega = \max\{ect_{\Omega_L} + p_{\Omega_R}, ect_{\Omega_R}\}$$

- With Ω_L, Ω_R two disjoint sets s.t. $\max\{r_i \mid i \in \Omega_L\} \leq \min\{r_i \mid i \in \Omega_R\}$



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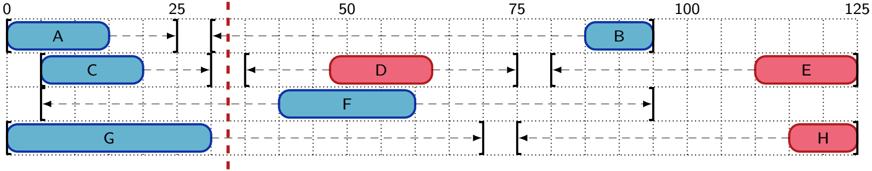

- Let $\Omega' \subseteq \Omega$ be such that $ect_\Omega = r_{\Omega'} + p_{\Omega'}$

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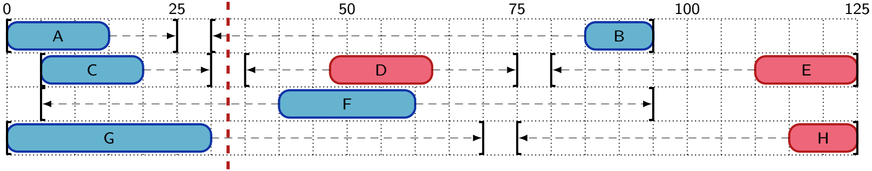

- Let $\Omega' \subseteq \Omega$ be such that $ect_\Omega = r_{\Omega'} + p_{\Omega'}$
  - Either $\Omega' \subseteq \Omega_R$ and therefore $ect_\Omega = ect_{\Omega'} = ect_{\Omega_R}$

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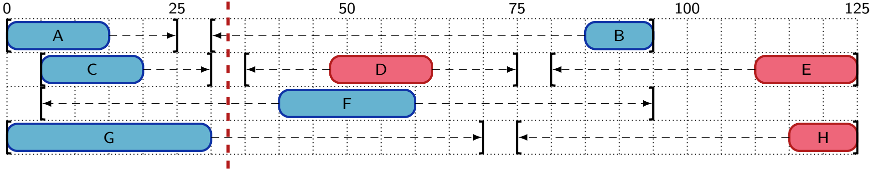

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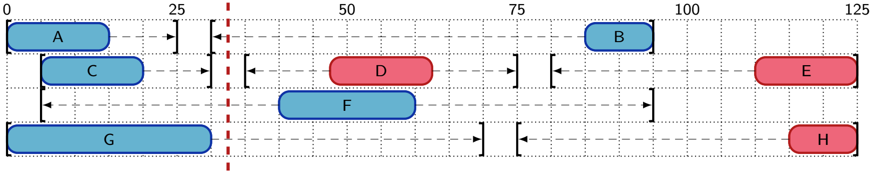

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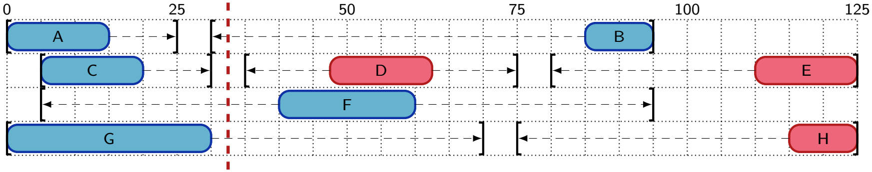

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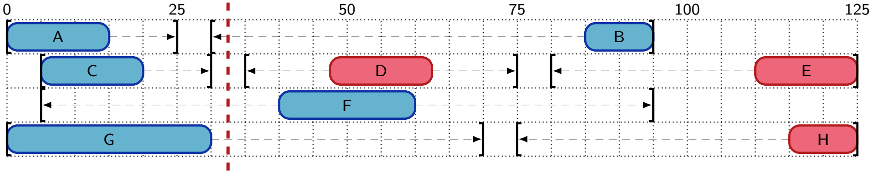

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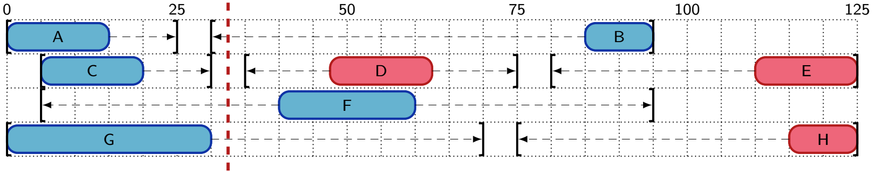

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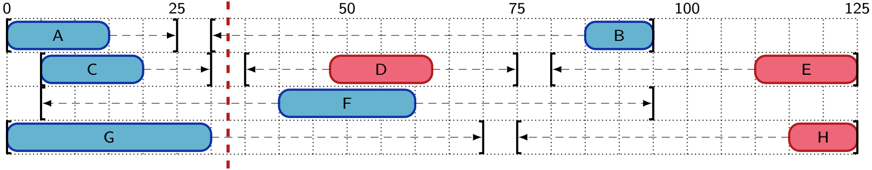

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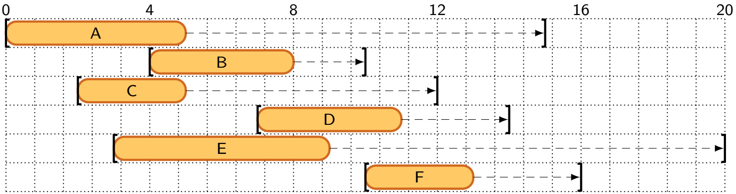
Overload Checking – Theta Tree

- Order the tasks by non-decreasing due date to compute $\mathcal{T}|_j$ for all $j \in \mathcal{T}$
- Order the tasks by non-decreasing release date to compute $ect_{\mathcal{T}|_j}$

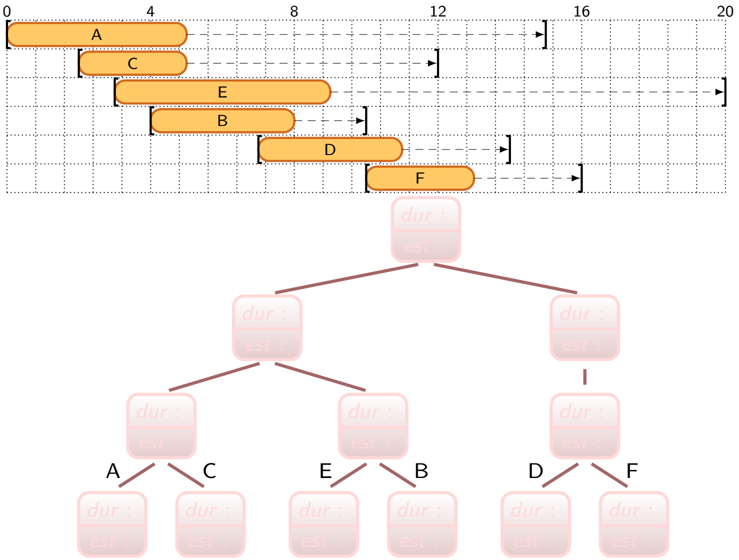
Solution

- Theta tree (Vilím, Barták, and Čepek, 2004)
  - Explore nested sets of tasks in any order (here non-decreasing due dates)
  - Incrementally compute a property (here $ect_{\mathcal{T}|_j}$) requiring another order

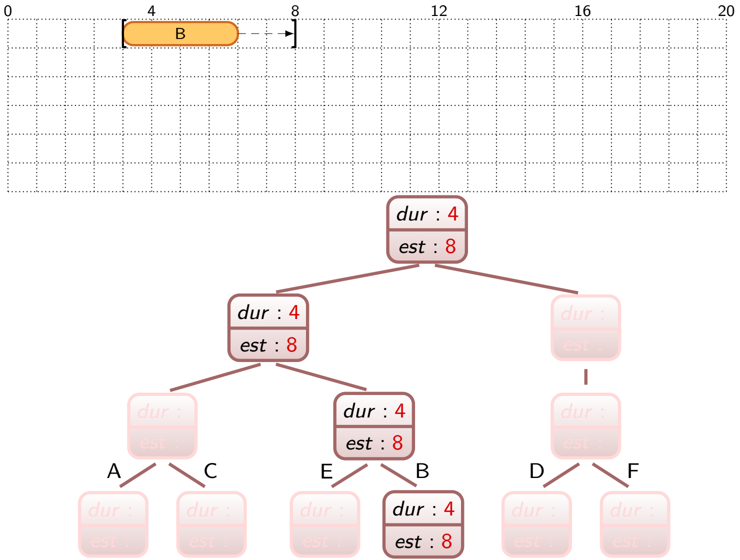
Theta Tree



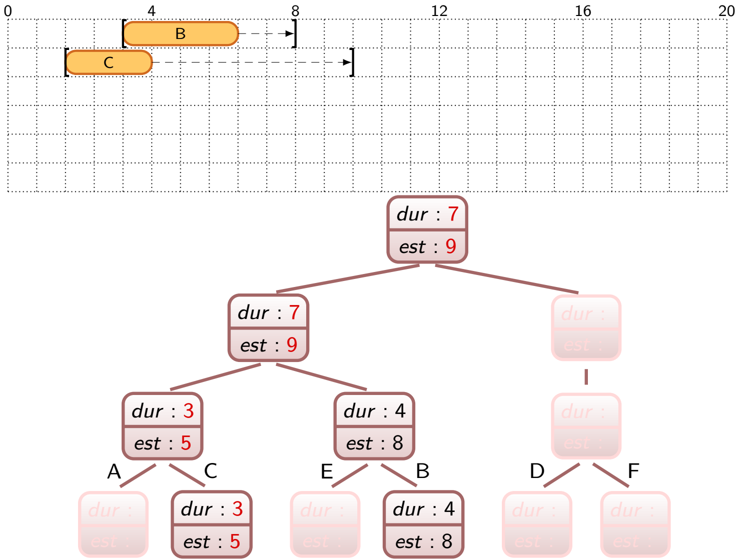
Theta Tree



Theta Tree



Theta Tree


0
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A
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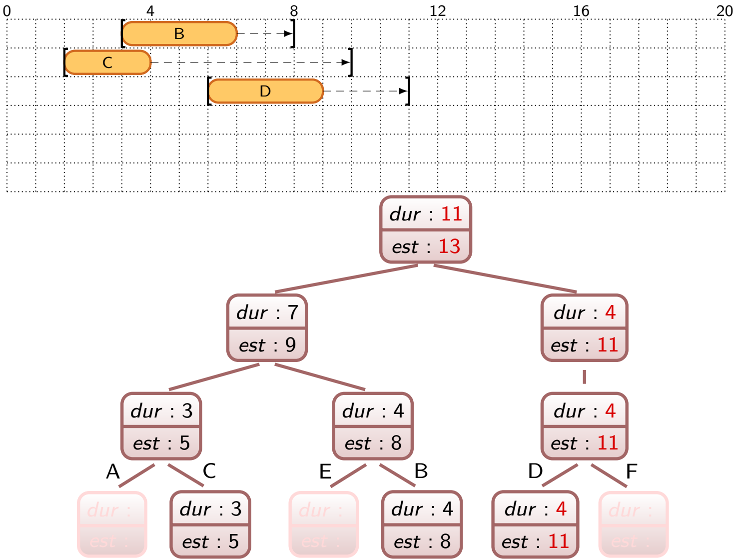
E
B
D
F
dur : 3
est : 5
dur : 4
est : 8

Theta Tree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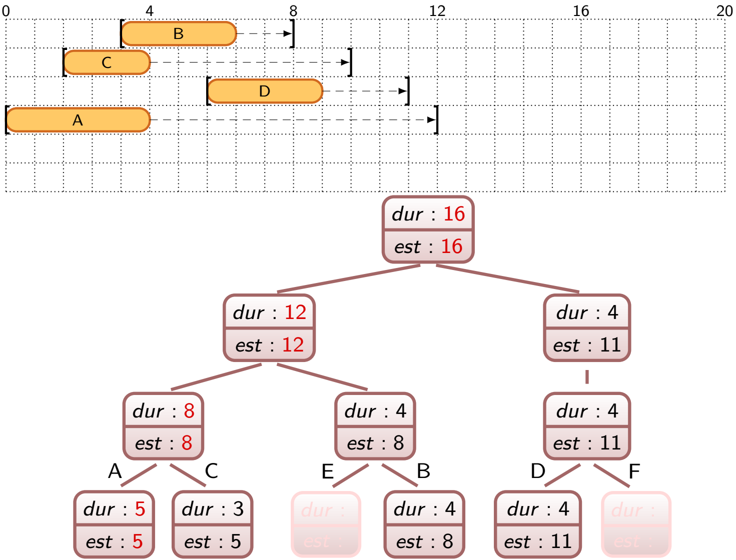
dur : 4
est : 11

Theta Tree



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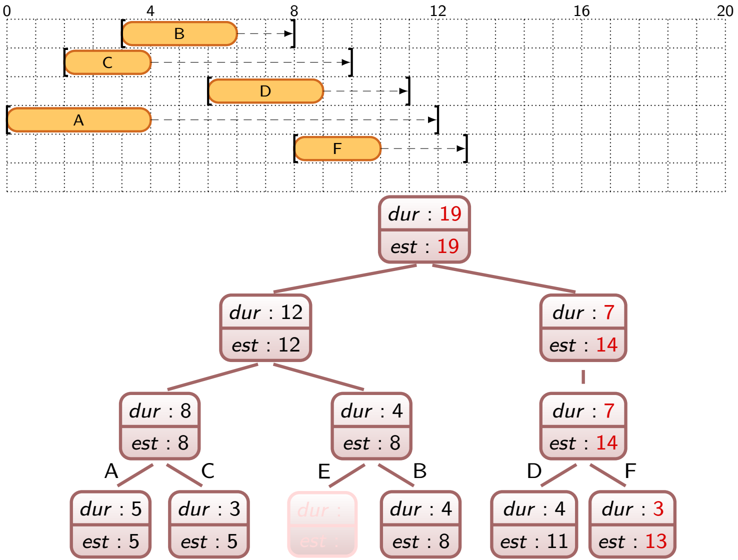
Theta Tree



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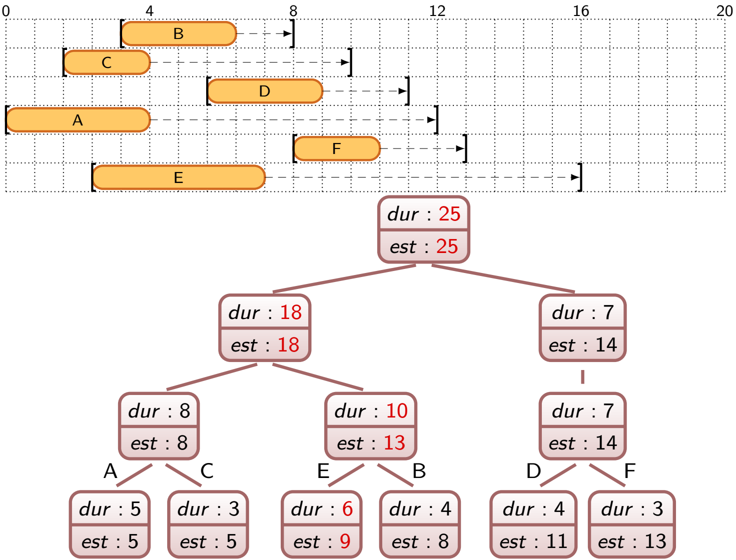
est : 13

Theta Tree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est : 25
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est : 18
dur : 7
est : 14
dur : 8
est : 8
dur : 10
est : 13
dur : 7
est : 14
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C
E
B
D
F
dur : 5
est : 5
dur : 3
est : 5
dur : 6
est : 9
dur : 4
est : 8
dur : 4
est : 11
dur : 3
est : 13

Edge finding

- $O(n \log n)$ algorithm based on Jackson Preemptive Schedule (Carlier and Pinson, 1994)

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- $O(n^2)$ algorithm easier to implement / better in practice (Martin and Shmoys, 1996) and (Baptiste, Le Pape, and Nuijten, 2001)

Single-machine algorithms

Edge finding

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- $O(n^2)$ algorithm easier to implement / better in practice (Martin and Shmoys, 1996) and (Baptiste, Le Pape, and Nuijten, 2001)
- Simpler algorithm in $O(n \log n)$ (Vilím, Barták, and Čepek, 2004)
  - Overload checking with theta tree, then mark the tasks as “maybe” one at a time

Single-machine algorithms

Edge finding

- $O(n \log n)$ algorithm based on Jackson Preemptive Schedule (Carlier and Pinson, 1994)
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Not first / Not last

- $O(n^2)$ algorithm (Le Pape and Baptiste, 1996)

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Not first / Not last

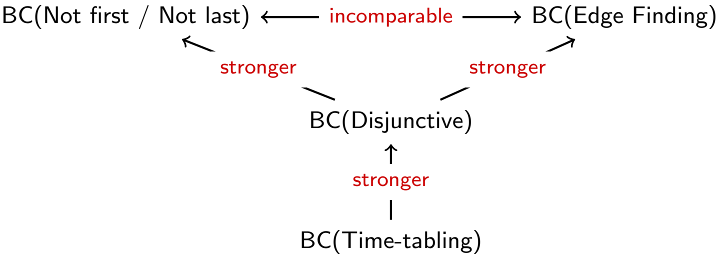
- $O(n^2)$ algorithm (Le Pape and Baptiste, 1996)
- Simpler algorithm in (Torres and Lopez, 2000)
- Theta-tree-based algorithm in $O(n \log n)$ (Vilím, 2004)

In the previous episode

- Time-tabling: usage profile $O(n)$ (Gay, Hartert, and Schaus, 2015)
- Disjunctive: either $i \prec j$ or $j \prec i$ $O(n^3)$ but incremental
- Edge finding, not-first, not-last: overload on Ω if we add i [not] firt/last $O(n \log n)$ (Vilím, Barták, and Čepek, 2004)

In the previous episode (single machine)

- Time-tabling: usage profile (good in the cumulative case) $O(n)$ (Gay, Hartert, and Schaus, 2015)
- Disjunctive: either $i \prec j$ or $j \prec i$ $O(n^3)$ but incremental
- Edge finding, not-first, not-last: overload on Ω if we add i [not] firt/last $O(n \log n)$ (Vilím, Barták, and Čepek, 2004)



Today

- Finish the review of resource propagation algorithms
  - Cumulative case
- Search
- Other types of resource

Cumulative Overload Checking

Notion of energy

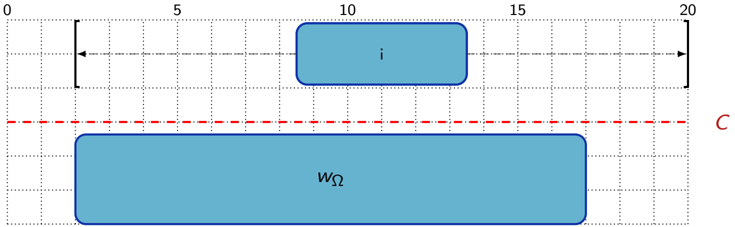
- $w_j = p_i c_i$
- $w_\Omega = \sum_{j \in \Omega} w_j$

- Overload checking is similar to the single-machine case

$$\forall \Omega \subseteq \mathcal{T} \quad w_\Omega \leq C(e_\Omega - s_\Omega)$$

Cumulative Edge Finding (Nuijten and Aarts, 1994)

$$\forall \Omega \subseteq \mathcal{T} \qquad w_\Omega \leq C(e_\Omega - s_\Omega)$$



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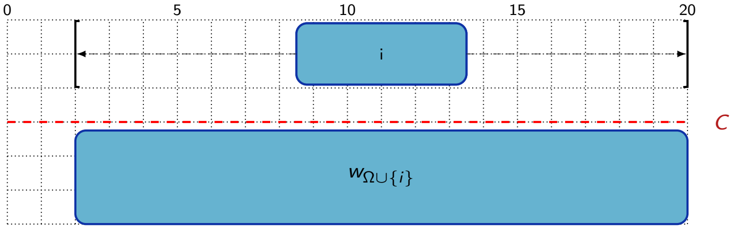
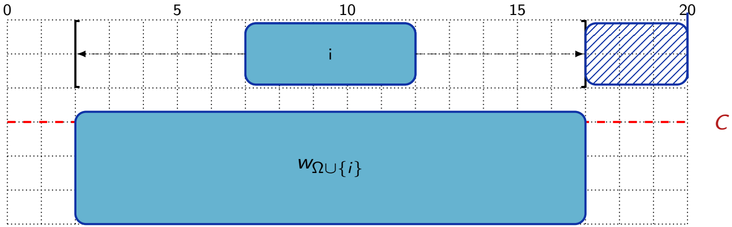


Cumulative Edge Finding (Nuijten and Aarts, 1994)

- If energy of Ω and i exceeds capacity when i is not last
  - then i has to be last

$$\forall \Omega \subseteq \mathcal{T} \qquad w_\Omega \leq C(e_\Omega - s_\Omega)$$

$$\forall \Omega \subseteq \mathcal{T},\ \forall i \notin \Omega \quad w_{\Omega \cup \{i\}} > C(e_\Omega - s_{\Omega \cup \{i\}}) \implies \Omega \prec i$$



Cumulative Edge Finding (Nuijten and Aarts, 1994)

- If energy of Ω and i exceeds capacity when i is not last
  - then i has to be last but not necessarily follows every task in Ω!

$$\forall \Omega \subseteq \mathcal{T} \qquad w_\Omega \leq C(e_\Omega - s_\Omega)$$

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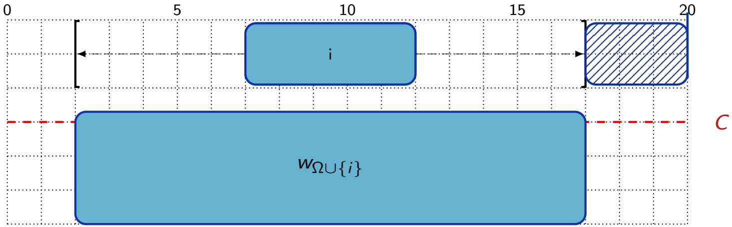

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$$\forall \Omega \subseteq \mathcal{T},\ \forall c \qquad R(\Omega, c) = w_\Omega - (C - c)(e_\Omega - s_\Omega)$$

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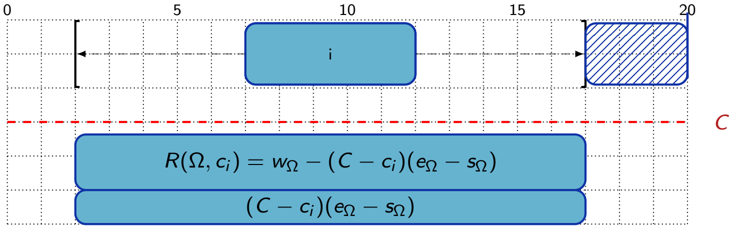

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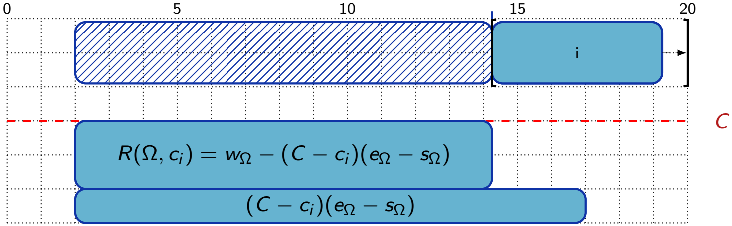

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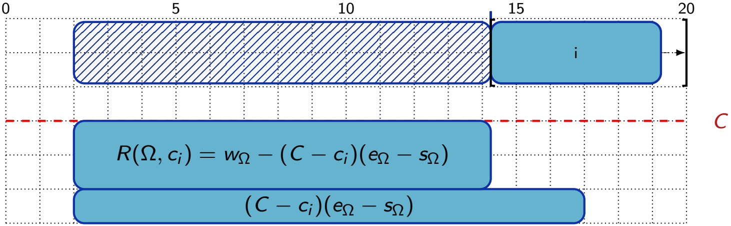

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- $O(n^2 k)$ (Nuijten and Aarts, 1994), $O(n^2)$ (Baptiste, Le Pape, and Nuijten, 2001), $O(kn \log n)$ (Vilím, 2009), $O(n^2)$ (Kameugne et al., 2011)

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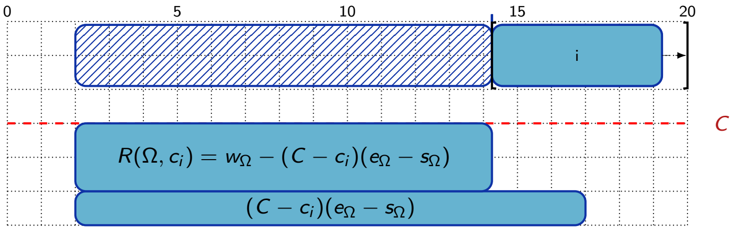

C. Extended Edge Finding (Nuijten and Aarts, 1994)

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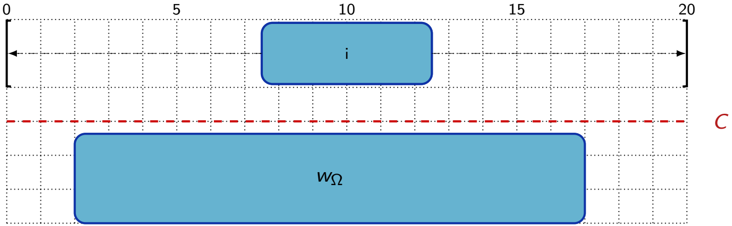

C. Extended Edge Finding (Nuijten and Aarts, 1994)

- Case where i may start before Ω but cannot be completed before Ω starts
  - Stronger constraint if we consider the overlap: $(e_i - s_\Omega)c_i$

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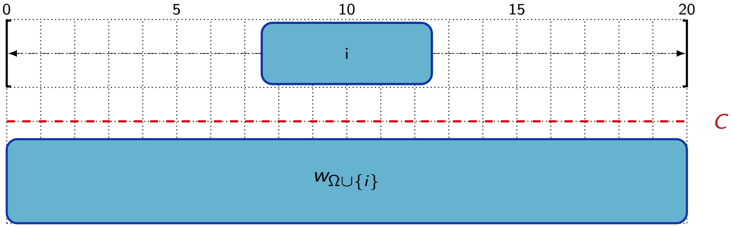

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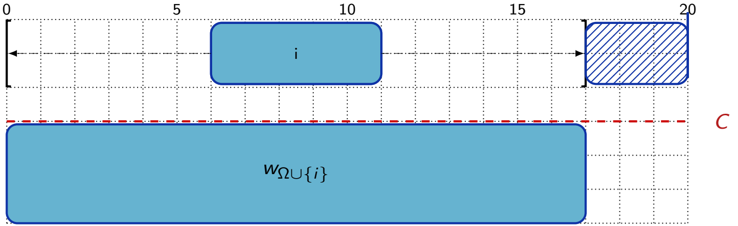

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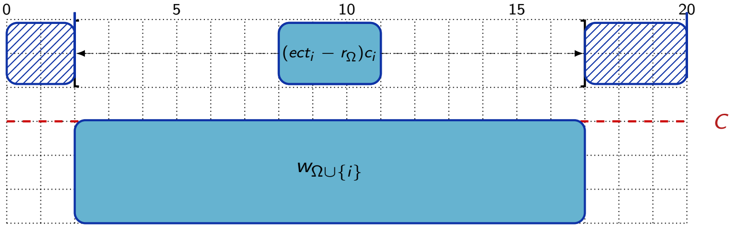

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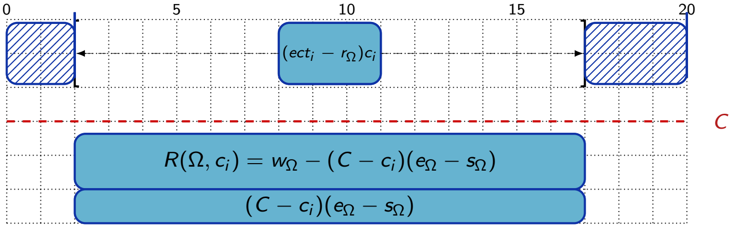

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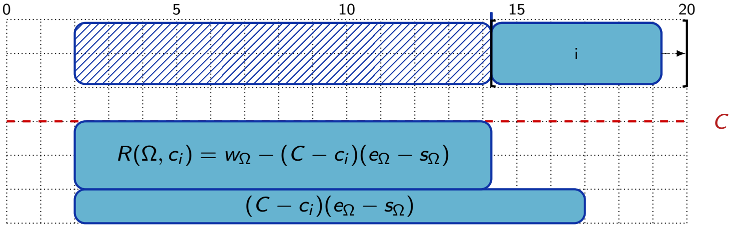

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- $O(n^3)$ (Nuijten and Aarts, 1994), $O(n^2)$ (Mercier and Hentenryck, 2008), $O(kn \log n)$ (Ouellet and Quimper, 2013)

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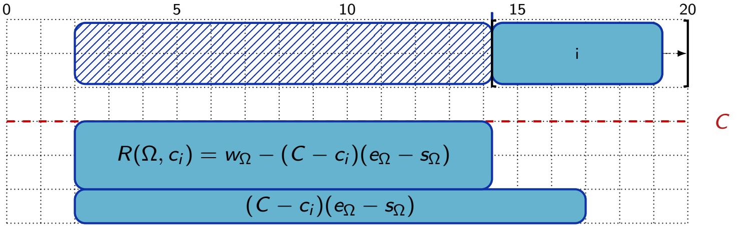
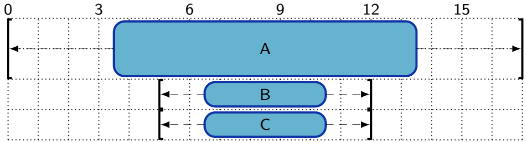


Time-tabling Edge Finding (Vilím, 2011)

- Time-tabling and (Extended) Edge finding are incomparable when $C > 1$



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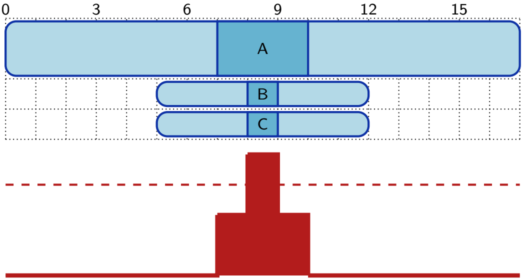

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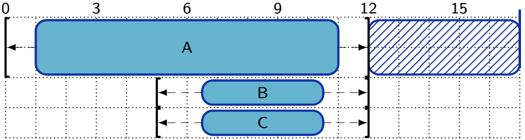

$$20 + 4 + 4 \leq 3 \times 12 = 36$$

Time-tabling Edge Finding (Vilím, 2011)

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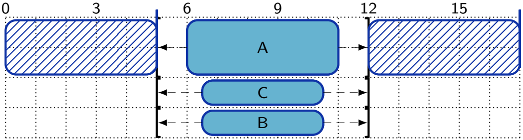
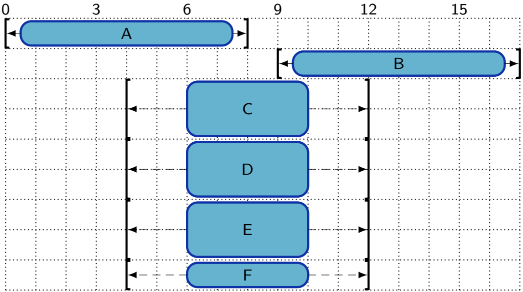


$$10 + 4 + 4 \leq 3 \times 7 = 21$$

Time-tabling Edge Finding (Vilím, 2011)

- Time-tabling and (Extended) Edge finding are incomparable when $C > 1$
  - Combine them! $3 \times 8 + 4 + 5 > 4 \times 8$



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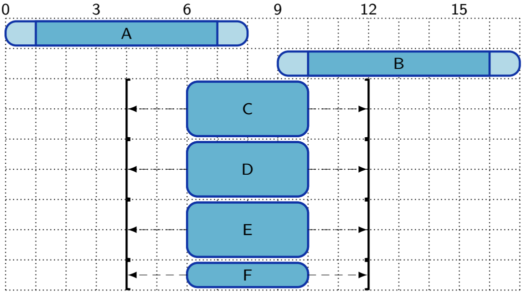

Time-tabling Edge Finding decomposition

- Channel Time-tabling and (Extended) Edge finding decompositions

$$
\begin{array}{rc}
\forall \Omega \subseteq \mathcal{T} & w_\Omega \leq C(e_\Omega - s_\Omega) \\
\forall \Omega \subseteq \mathcal{T},\ \forall c & R(\Omega, c) = w_\Omega - (C - c)(e_\Omega - s_\Omega) \\
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\end{array}
$$

Time-tabling Edge Finding decomposition

- Channel Time-tabling and (Extended) Edge finding decompositions

$$
\begin{array}{rl}
\forall \Omega \subseteq \mathcal{T} & w_\Omega \leq C(e_\Omega - s_\Omega) - \sum_{t=s_\Omega}^{e_\Omega} \sum_{i \notin \Omega} c_i a_i^t \\
\forall \Omega \subseteq \mathcal{T},\ \forall c & R(\Omega, c) = w_\Omega - (C - c)(e_\Omega - s_\Omega) \\
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\forall i \forall t & a_i^t \iff s_i \leq t \wedge t < e_i
\end{array}
$$

Time-tabling Edge Finding decomposition

- Channel Time-tabling and (Extended) Edge finding decompositions
  - Algorithm in $O(n^2)$ for Edge finding + Time-tabling (Vilím, 2011)
  - Algorithm in $O(kn\log n)$ for Edge finding + Extended Edge finding + Time-tabling (Ouellet and Quimper, 2013)

$$\begin{aligned}
\forall \Omega \subseteq \mathcal{T} &\quad w_\Omega \leq C(e_\Omega - s_\Omega) - \textstyle\sum_{t=s_\Omega}^{e_\Omega} \sum_{i \notin \Omega} c_i a_i^t \\
\forall \Omega \subseteq \mathcal{T},\ \forall c &\quad R(\Omega, c) = w_\Omega - (C - c)(e_\Omega - s_\Omega) \\
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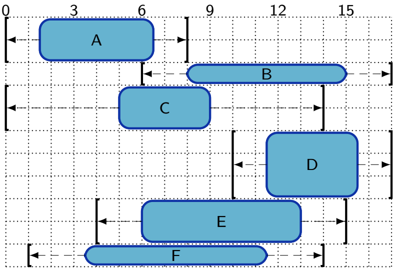
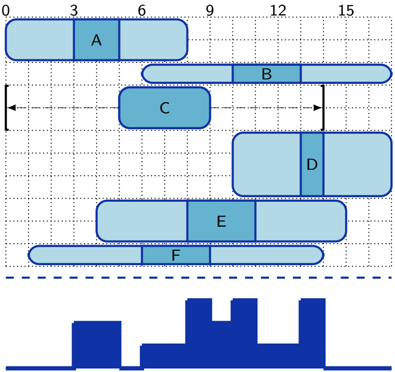
\end{aligned}$$

Time-tabling Edge Finding algorithm

Algorithm of (Ouellet and Quimper, 2013)



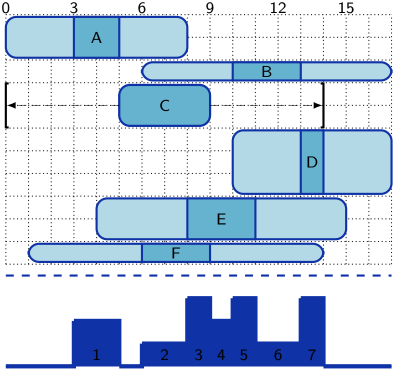
Algorithm of (Ouellet and Quimper, 2013)

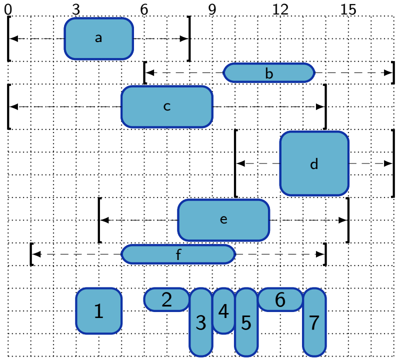


Time-tabling Edge Finding algorithm

Algorithm of (Ouellet and Quimper, 2013)

- A fixed task for each interval of the profile, remove the compulsory part from the task
- Apply any extended edge finding algorithm

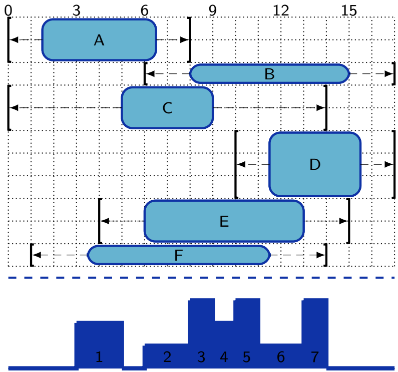


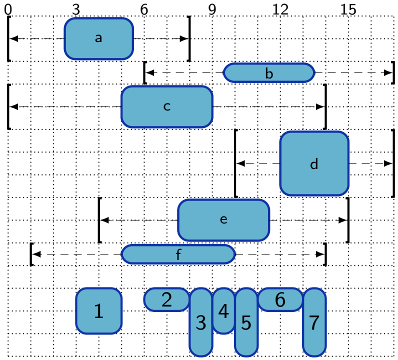


Time-tabling Edge Finding algorithm

Algorithm of (Ouellet and Quimper, 2013)

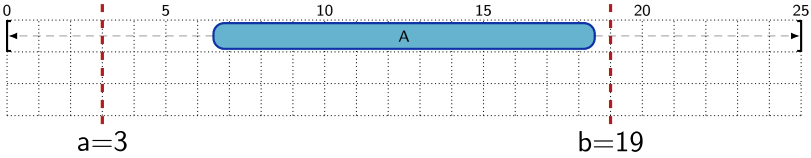
- A fixed task for each interval of the profile, remove the compulsory part from the task
- Apply any extended edge finding algorithm





Energetic reasoning (Lopez, 1991)

- Energy of a set of tasks $\Longrightarrow$ Energy of all tasks over a given interval



Notion of energy (over an interval)

- $p_j^+(a) = \max(0, p_j - \max(0, a - s_j))$
- $p_j^-(b) = \max(0, p_j - \max(0, e_j - b))$
- $w_j(a, b) = c_j \min(b - a, p_j^+(a), p_j^-(b))$
- $W(a, b) = \sum_{j \in \mathcal{T}} w_j(a, b)$

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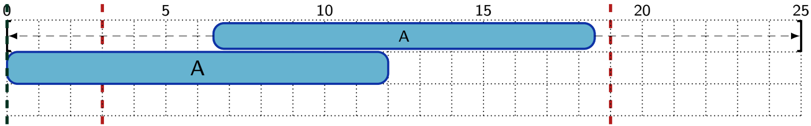

$r_A = 0$ a=3 b=19

$\longleftarrow 3 \longrightarrow$

$p_A^+(3) = 12 - 3 = 9$

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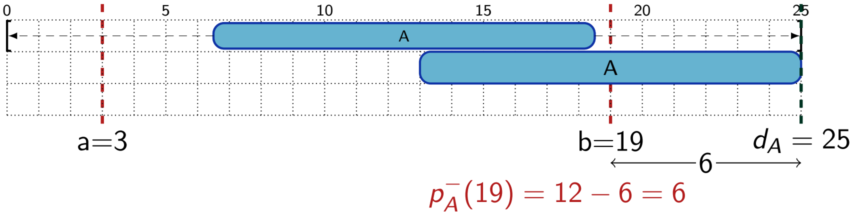

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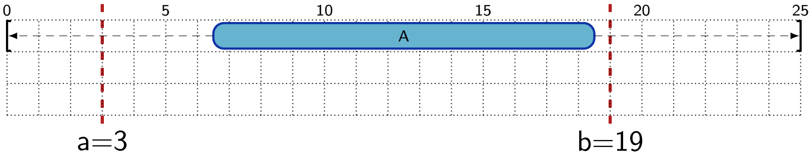

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Energetic reasoning decomposition

$$\forall 0 \leq a < b < h \quad W(a, b) \leq C(b - a) \quad \text{(ER)}$$

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Relevant intervals (Baptiste, 1998)

$$\begin{aligned} O_1(i) &= \{r_i, lst_i, ect_i\} \\ O_2(i) &= \{d_i, lst_i, ect_i\} \\ O(i,t) &= \{lst_i - t \mid i \in \mathcal{T}\} \end{aligned}$$

- (ER) holds iff it holds for every $i \neq j$ and every $a < b$ s.t.
  - $a, b \in O_1(i) \times O_2(j)$
  - $a, b \in O_1(i) \times O(j, a)$
  - $a, b \in O_2(i) \times O(j, b)$

- $O(n^2)$ relevant intervals, 15 for each pair i, j (Baptiste, Le Pape, and Nuijten, 2001)
  - Algorithm to check them all in $O(n^2)$ (Baptiste, Le Pape, and Nuijten, 2001)

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- Possible with only 2 intervals for each pair (Derrien and Petit, 2014)

Energetic reasoning decomposition

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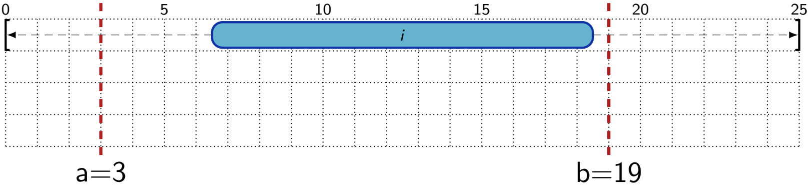

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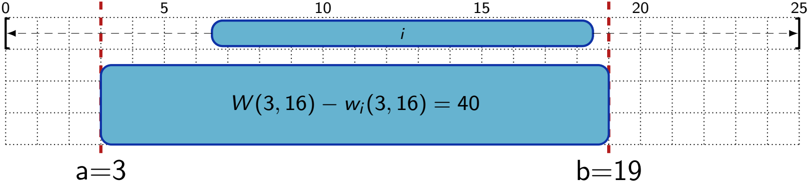
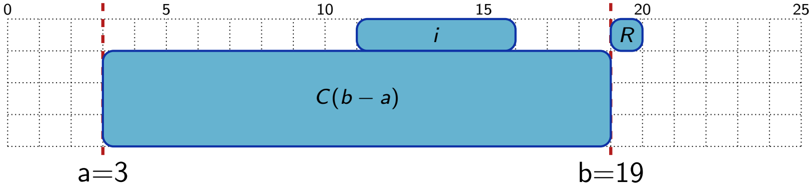


- $C = 3$, maximum energy on $[3,19) = 48$ $(W(3,16) = 46,\ w_i(3,16) = 6)$

Energetic reasoning decomposition

$$\forall 0 \leq a < b < h \quad W(a,b) \leq C(b-a)$$

$$\underbrace{W(a,b) + p_i^+(a) - w_i(a,b) - C(b-a)}_{R \text{ (at least after } b)} > 0 \implies e_i \geq b + \frac{R}{c_i}$$



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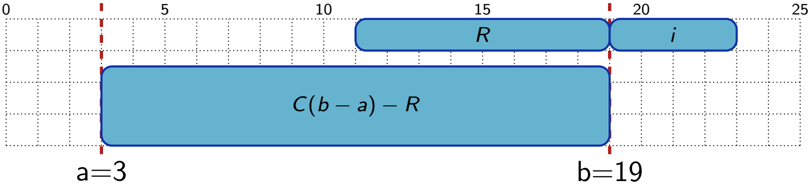
$$\min(b-a, p_i^+(a)) - \underbrace{w_i(a,b) + C(b-a) - W(a,b)}_{R \text{ (at most before } b)} > 0 \implies s_i \geq b - \frac{R}{c_i}$$



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LAAS CNRS

Energetic reasoning algorithm

- $O(n^3)$ (Baptiste, Le Pape, and Nuijten, 2001)

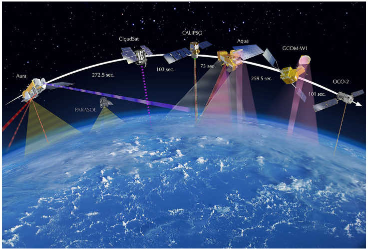
Energetic reasoning algorithm

- $O(n^3)$ (Baptiste, Le Pape, and Nuijten, 2001)
- $O(n^2 \log n)$ (Bonifas, 2014; Tesch, 2016)
  - Not complete, but at least one bound adjustment (hence $O(kn^2 \log n)$ where k is the number of tasks requiring a bound adjustment)

Part II: Search

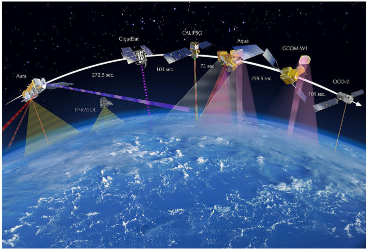
The importance of search: a short story

- Jobs: Files to transfer
- Resources:
  - Download channels: at most that many simultaneous downloads
  - Memory banks: cannot download two files stored on the same memory bank simultaneously
- Download as much data as possible within a given time window



The importance of search: a short story

- Jobs: Files to transfer
- Resources:
  - Download channels: at most that many simultaneous downloads
    - Cumulative resource shared by every task
  - Memory banks: cannot download two files stored on the same memory bank simultaneously
    - Tasks partitioned in as many unary resources as memory banks (m)
- Download as much data as possible within a given time window
  - Minimize makespan

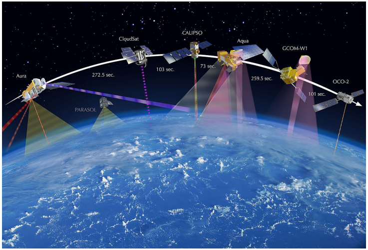


The importance of search: a short story

- Alas, our method was hardly better than a very basic greedy algorithm...

Greedy algorithm

- **Repeat:**
  - Choose the largest task a from the resource with highest demand
  - Schedule a as soon as possible



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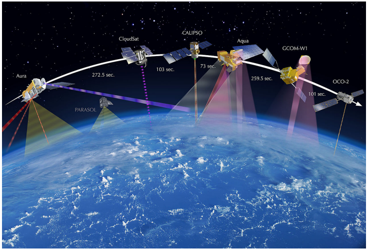

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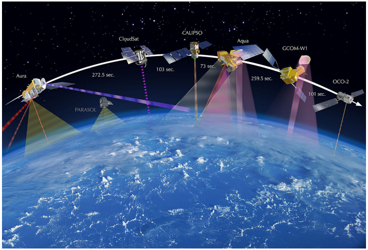

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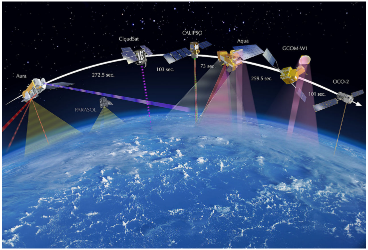
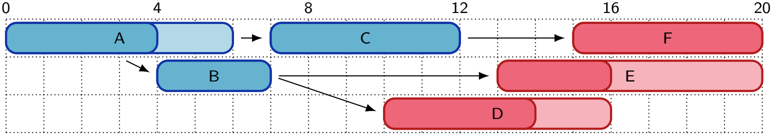
Not all resource scheduling are hard!

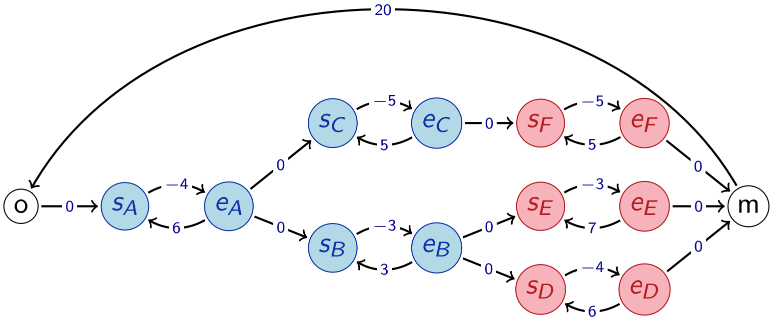


Search strategy

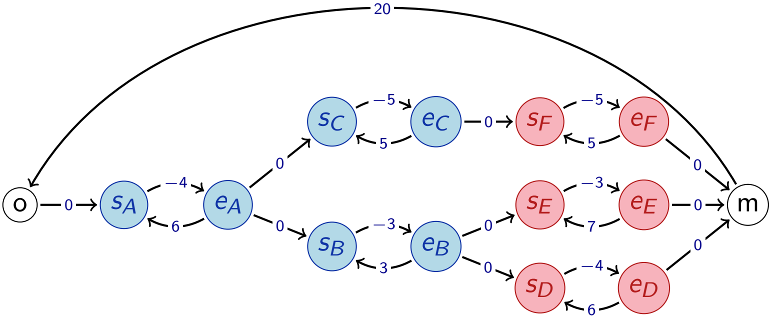
- Default CP strategy is a very bad idea:
  - Choose task i minimizing $d_i - r_i - p_i$ (and/or most constrained by resources and precedences)
  - Branch on $s_i = r_i$ or $s_i > r_i$: create "holes", dependent on the precision
- "Schedule or postpone" $\simeq$ branch on the task to start at the first available slot
  - Circumvent the precision problem, usually efficient for makespan minimization
  - Non trivial to implement in a classical CP solver
  - Might be incomplete for some objectives or constraints
- Sophisticated techniques in the latest CP solver (CP Optimizer) (Vilím, Laborie, and Shaw, 2015)
  - Alternate between Large Neighborhood Search and "Failure Directed Search"

Precedence graph / Difference system





Precedence graph as "domain"



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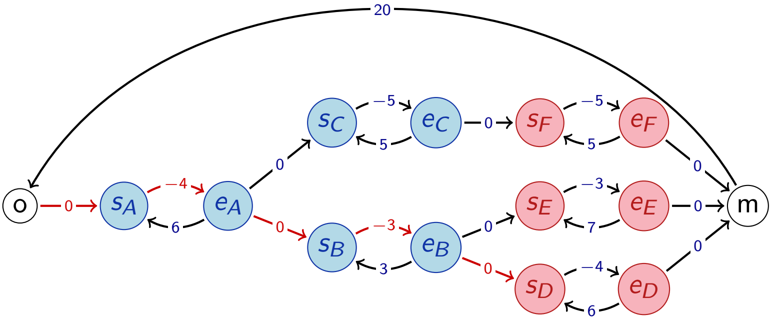

- Lower bound of node x is $-p_{0,x}$ where $p_{0,x}$ is the shortest path from 0 to x

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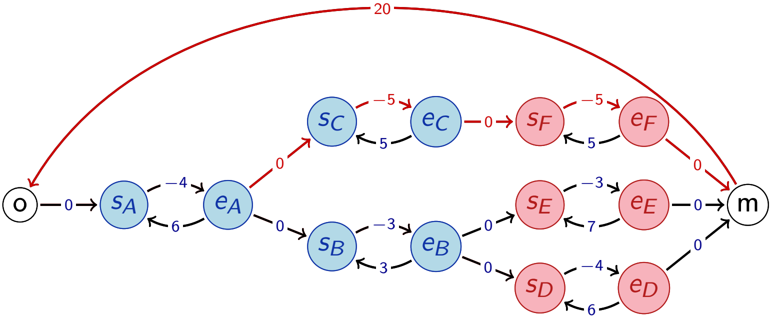

- Lower bound of node x is $-p_{0,x}$ where $p_{0,x}$ is the shortest path from 0 to x
- Upper bound of node x is $p_{x,0}$

Precedence graph as "domain"

- Convenient way to implement the domain / solution space
  - Fewer concepts (nodes, arcs vs. min/max duration p_i, release and due dates r_i, d_i, precedences)
  - Clean propagation
    - Lower, upper bounds and negative cycles: [Bellman–Ford]
    - Transitive closure on precedences: [Floyd–Warshall]

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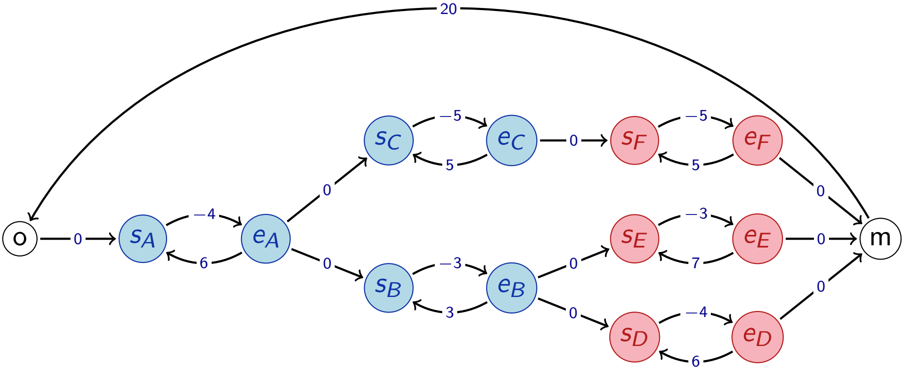
- Simulated by precedence constraints propagation and the "orchestra's conductor"

Search on the precedence graph

- Variable b_{ij} standing for $i \prec j$ for each pair of tasks i, j sharing a resource



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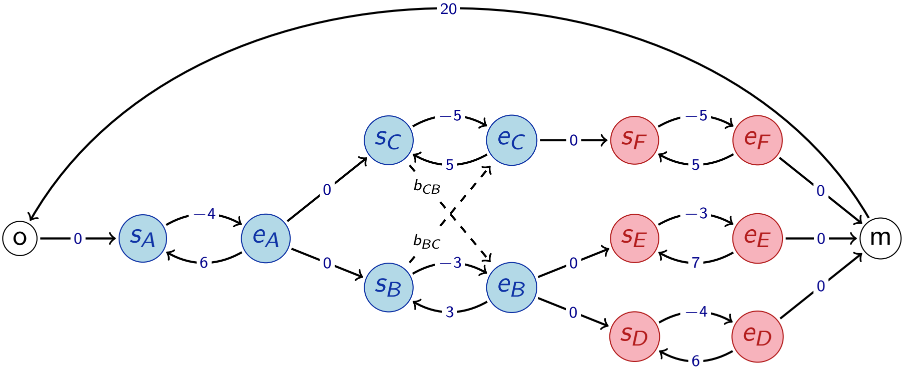

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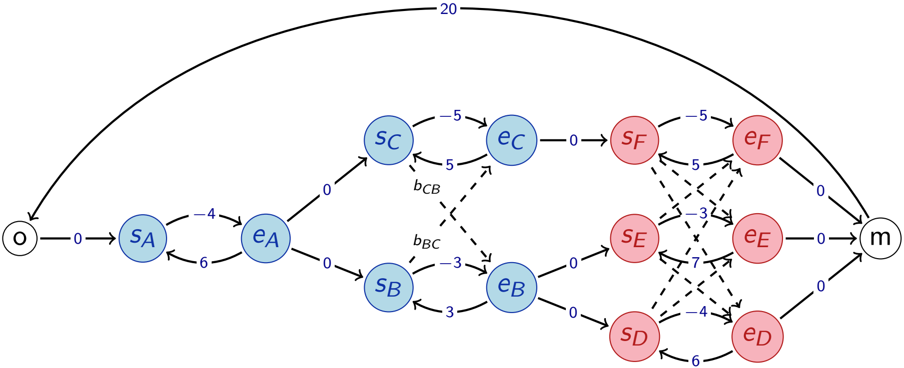
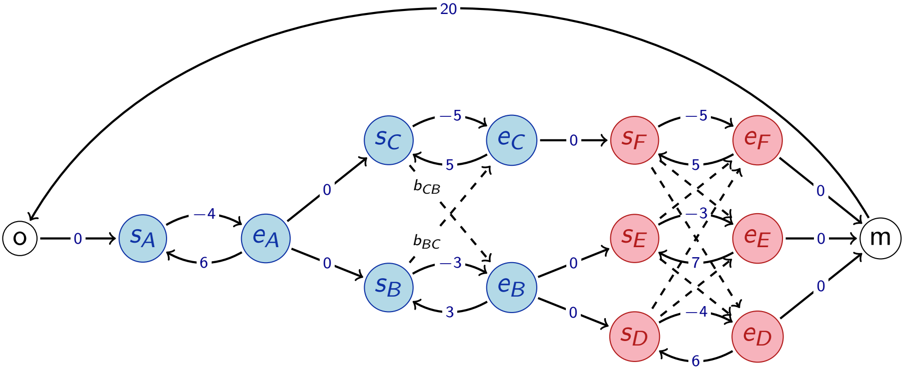


Search on the precedence graph

- Variable b_{ij} standing for $i \prec j$ for each pair of tasks i, j sharing a resource
  - Disjunctive constraints $\implies$ there is a solution iff there is no negative cycle



Conflict directed scheduling (Grimes and Hebrard, 2015)

Conflict weighting

- Only disjunctive constraints, one Boolean variable for each
- Weighted Degree: choose the tasks involved in the most conflicts
- Branch on the Boolean variables (post precedence one way)

IBM CP Optimizer

- Every algorithm seen here (use Cplex's linear relaxation?)
- Alternate large neighborhood search and Failure Directed Search
- Specialized strategies and auto-tuning

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	Objective	Proof	Objective	Proof
C_{max}	0.50%	**75%**	**0.03%**	56%

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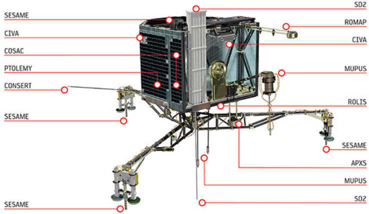
$T_{\sum}$	**1.59%**	**73%**	2.52%	33%

Part III: Other types of resource

Planning the mission of Philae on the comet 67P

(Simonin et al., 2015)

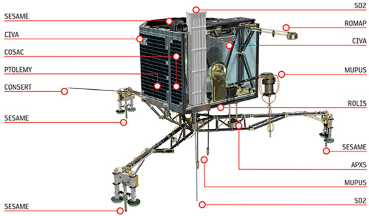
- Jobs: Scientific experiments
- Resources:
  - Batteries: threshold on the instant energy consumption
  - Memory: experiments produce data and transfers are possible only when Rosetta is visible



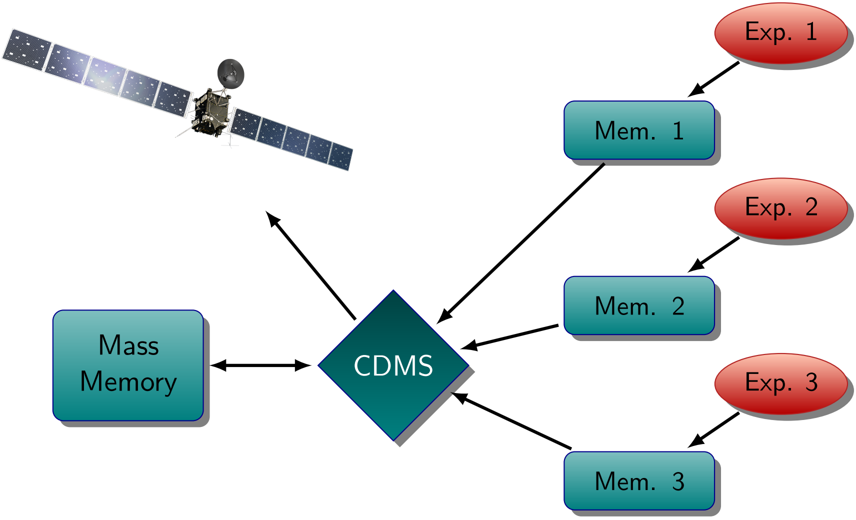
Planning the mission of Philae on the comet 67P

(Simonin et al., 2015)

- Jobs: Scientific experiments
- Resources:
  - Batteries: threshold on the instant energy consumption
    - ★ Nested cumulative constraints
  - Memory: experiments produce data and transfers are possible only when Rosetta is visible
    - ★ Memory / transfer channel resources (?)

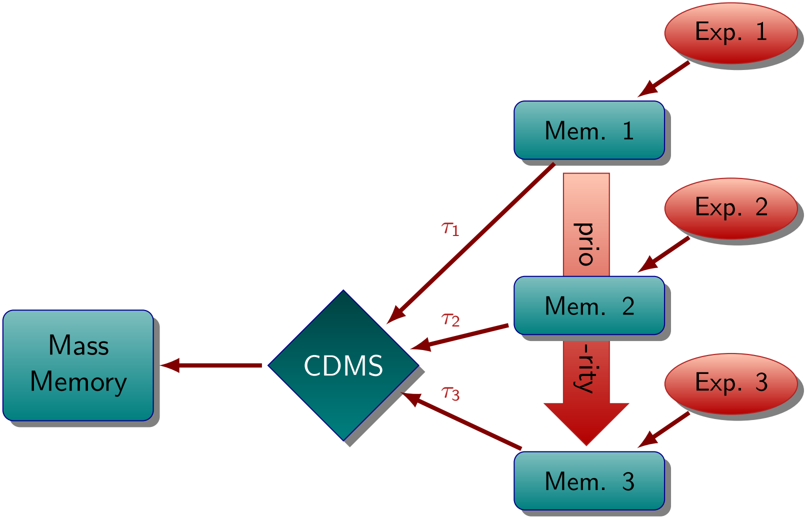


Command and Data Management Subsystem



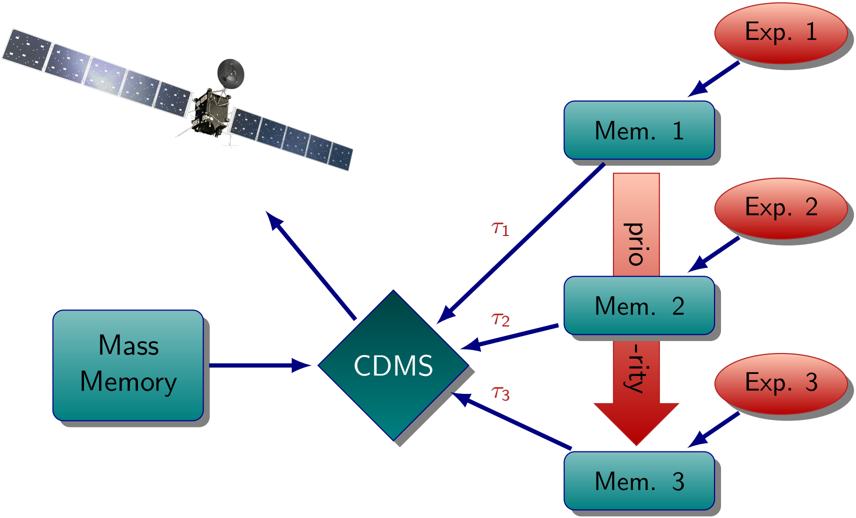
Command and Data Management Subsystem

(non-visibility)



Command and Data Management Subsystem

(visibility)



Simulating the Transfers

Original implementation

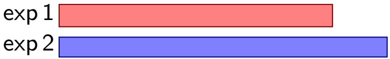
`IlcReservoir` constraints and optional transfer tasks

Simulating the Transfers

Original implementation

`IlcReservoir` constraints and optional transfer tasks

- Two experiments producing data
  - Exp.2 has higher priority
  - production rate $<$ transfer rate



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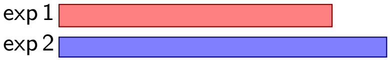
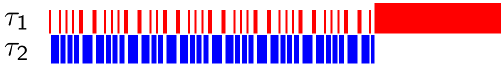


- Transfers
  - Switch back and forth from Exp.2 to Exp.1

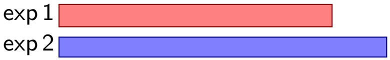


Simulating the Transfers

Original implementation

`IlcReservoir` constraints and optional transfer tasks

- Two experiments producing data
  - Exp.2 has higher priority
  - production rate < transfer rate



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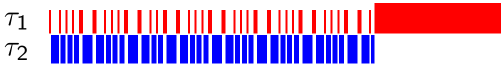
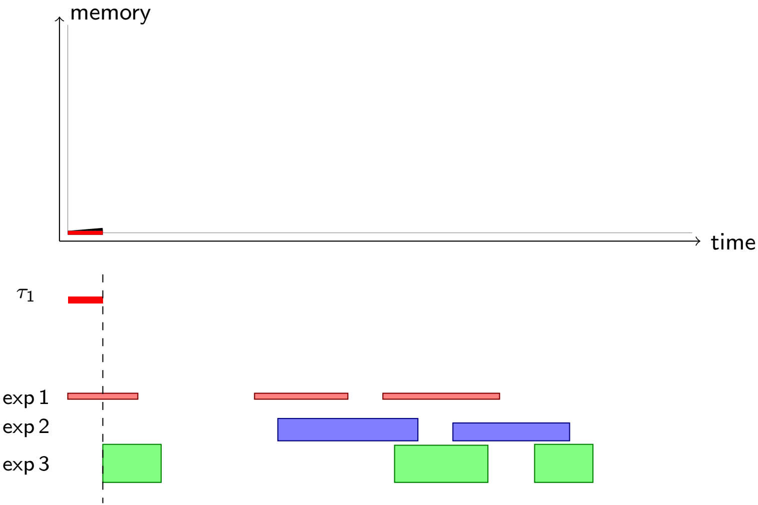


- Modeled as bandwidth sharing
  - Depends on priority, production and transfer rates

Checking the Constraint



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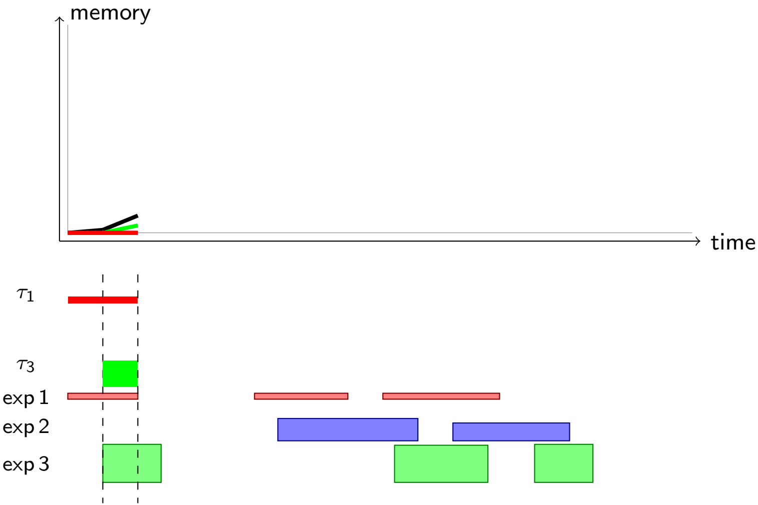

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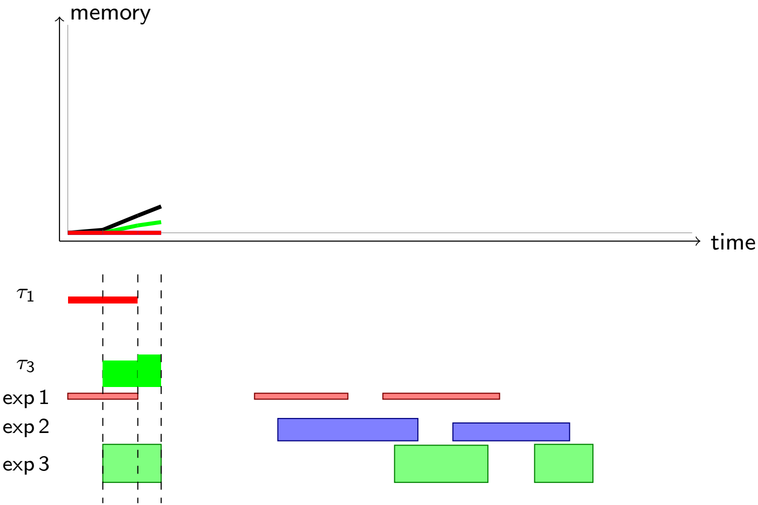

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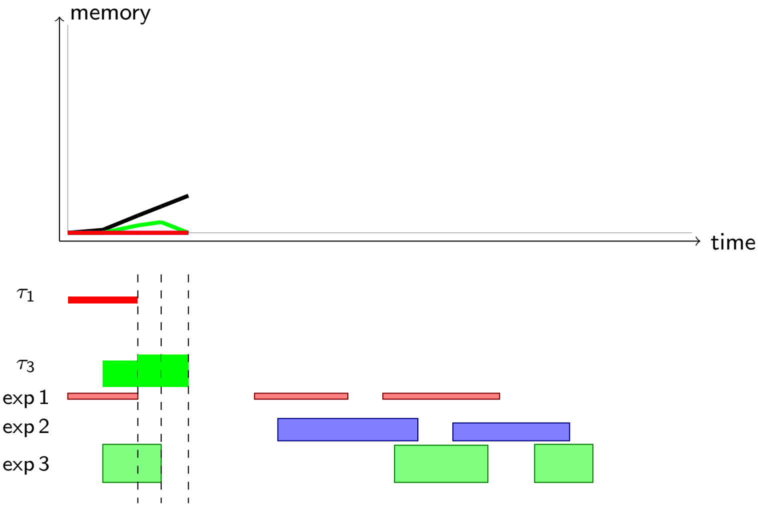

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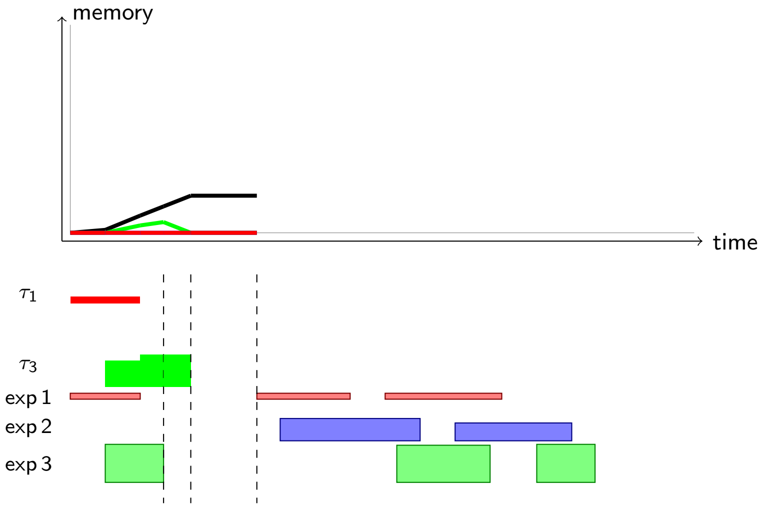

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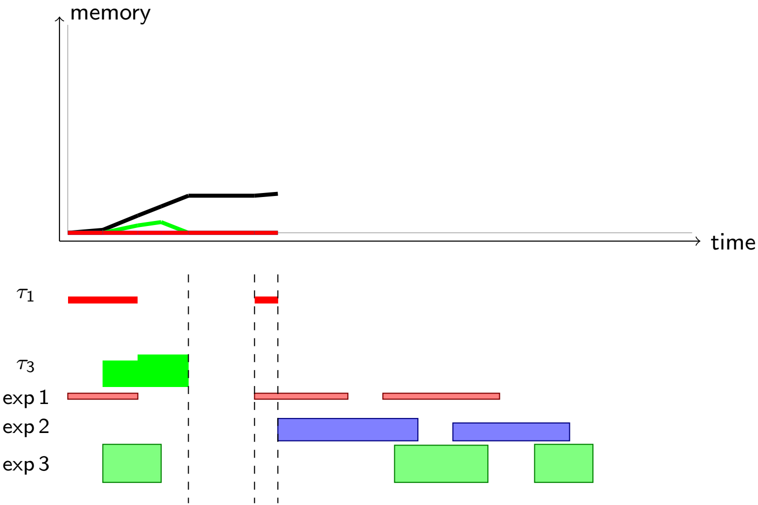

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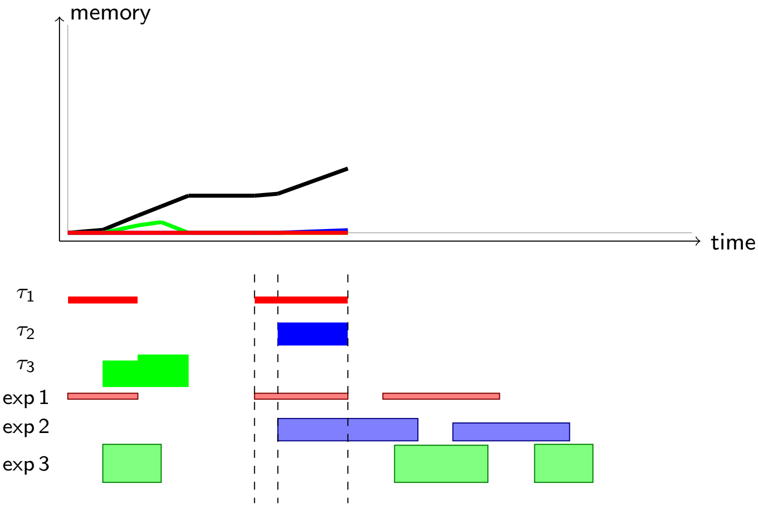

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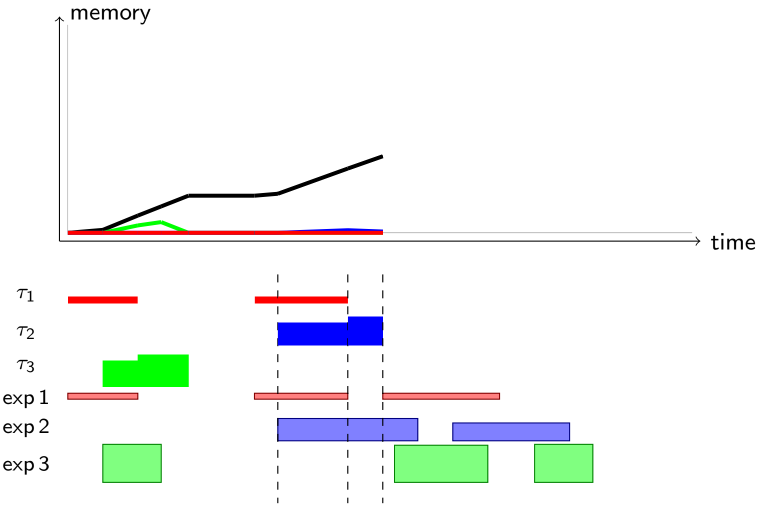

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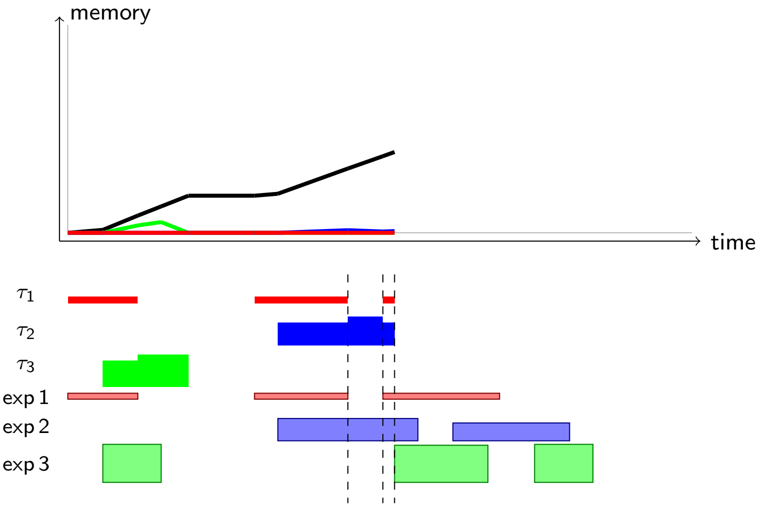

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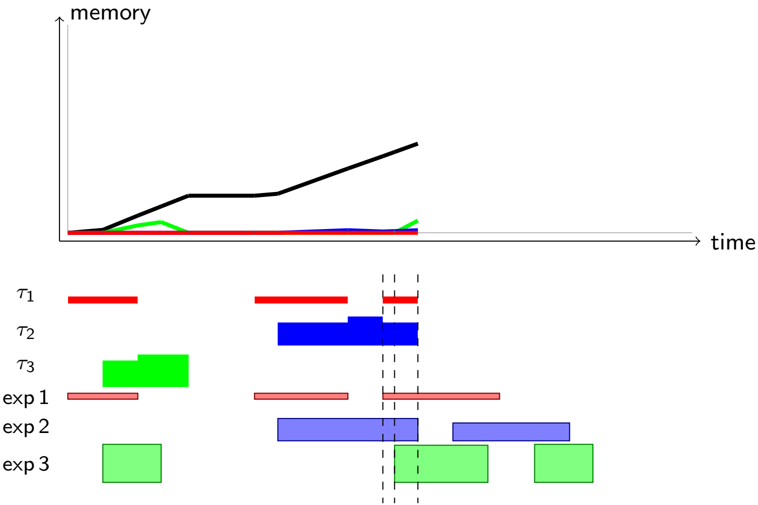

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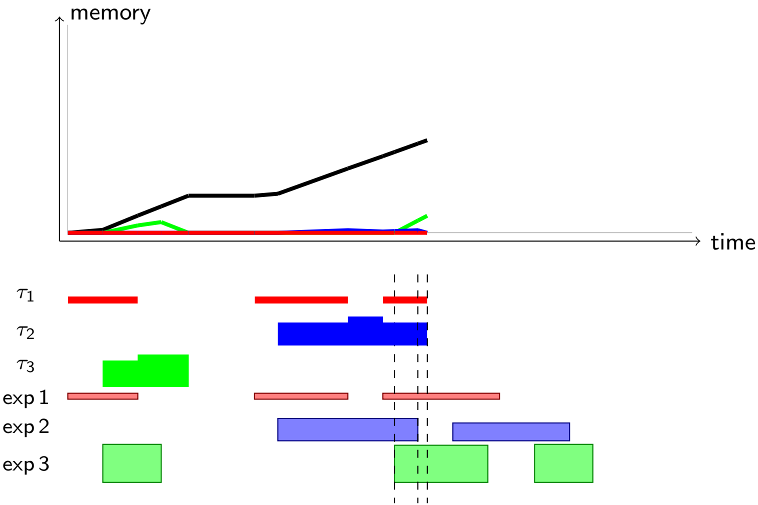

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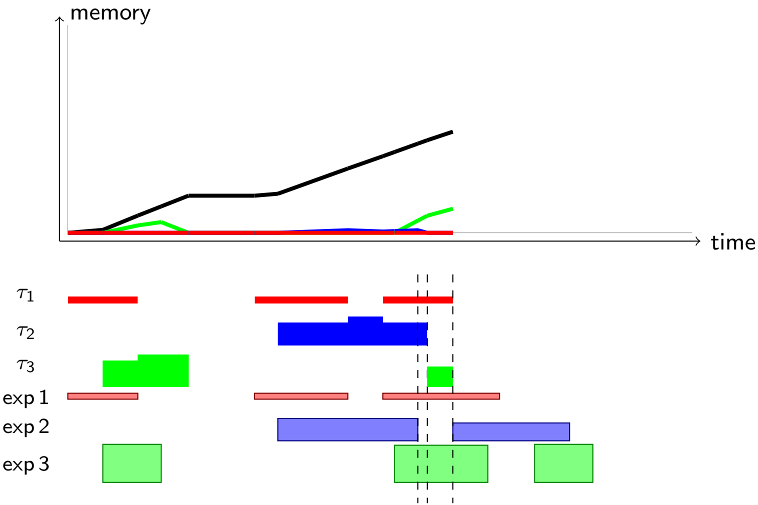

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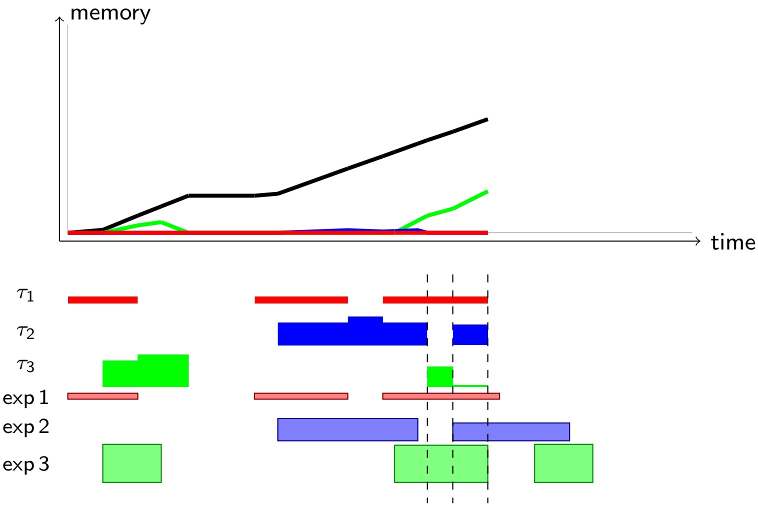

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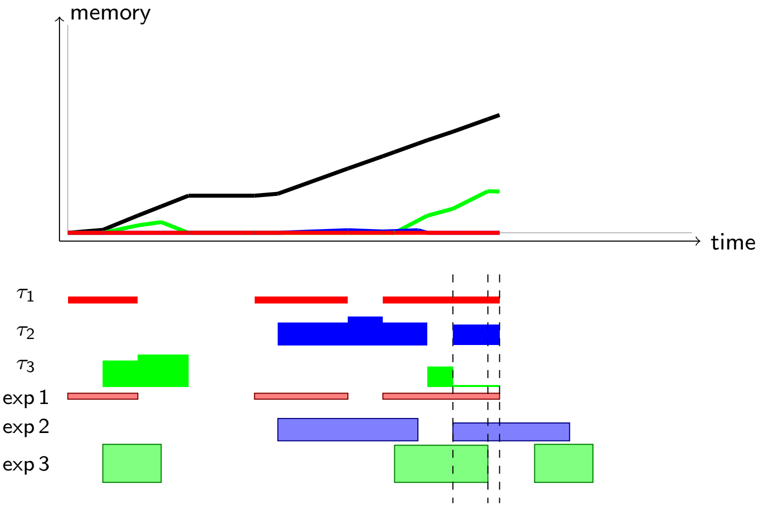

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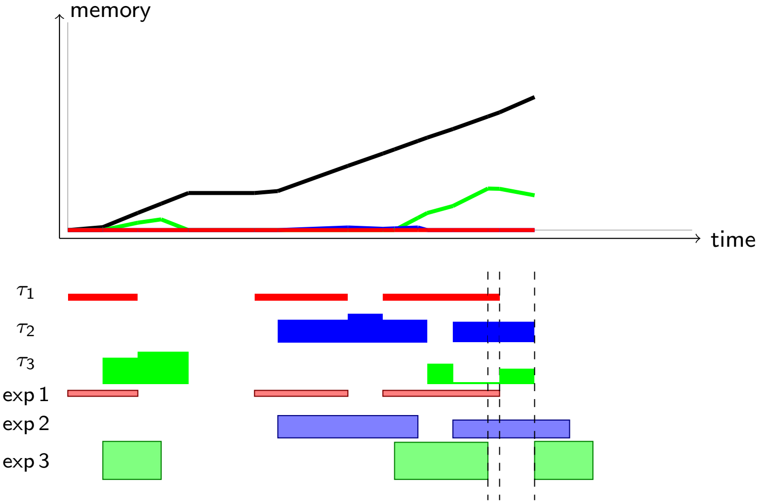

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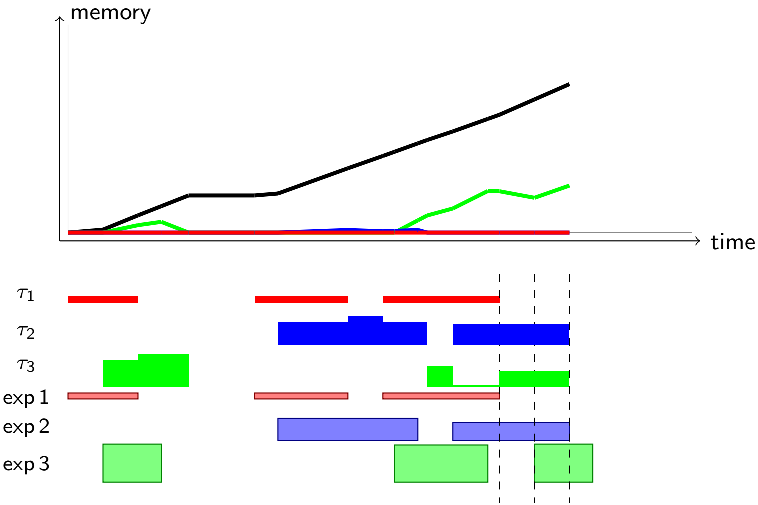

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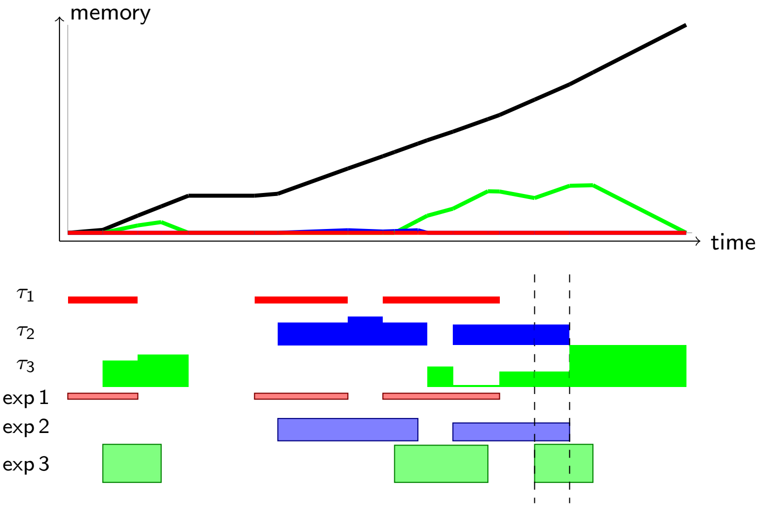

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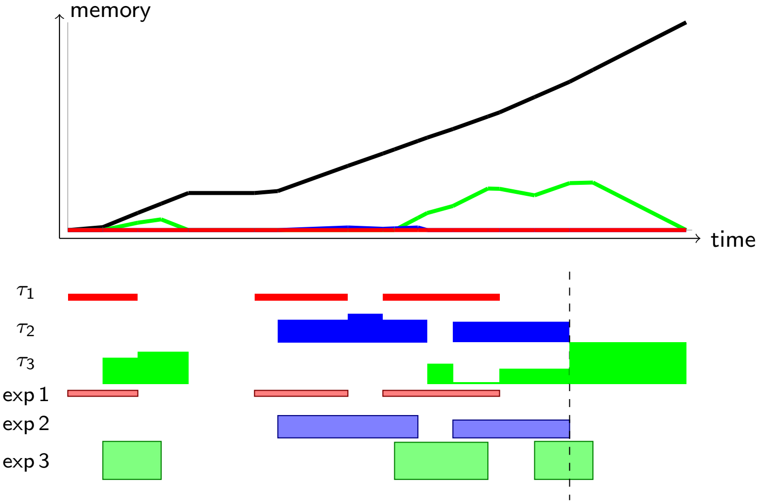

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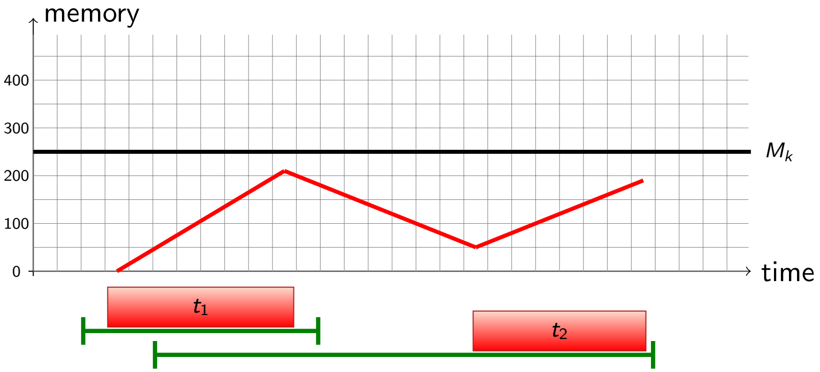
- Bound adjustments? Two principles:
  - Producing too much data too quickly can lead to data loss
  - Filling up the mass memory while not in visibility can lead to data loss

Propagation: production/transfer rate

For a set of tasks

- lower bound on how much time the CDMS needs to transfer without data loss



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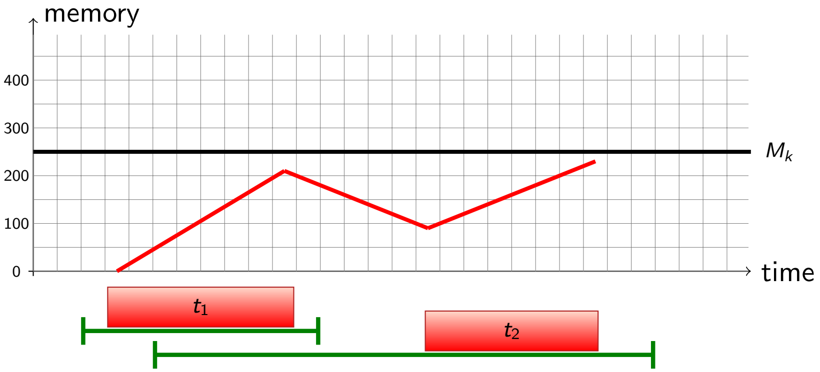

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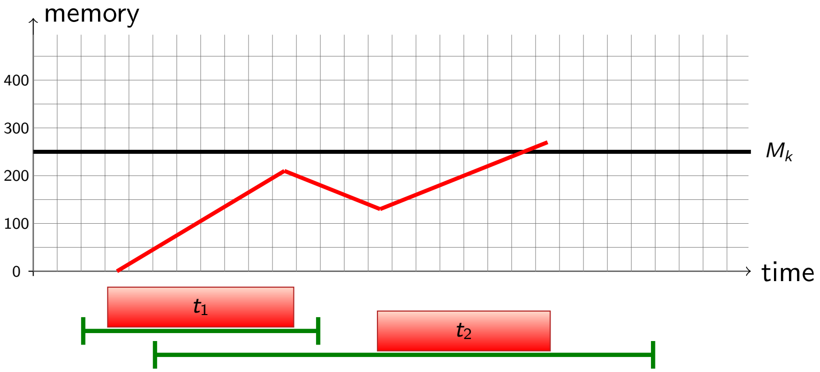

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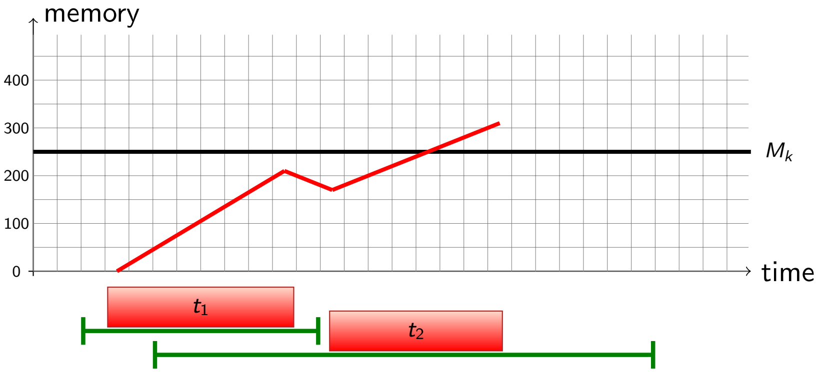

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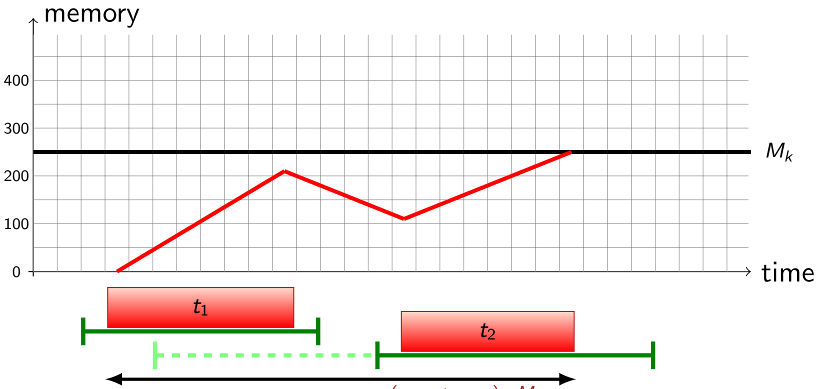

$$end(t_2) - start(t_1) \geq \frac{(\pi_1 p_1 + \pi_2 p_2) - M_k}{\tau}$$

Propagation: production/transfer rate

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- Minimum amount of higher priority data to transfer on $[a, b]$ over transfer rate:
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Filtering rule

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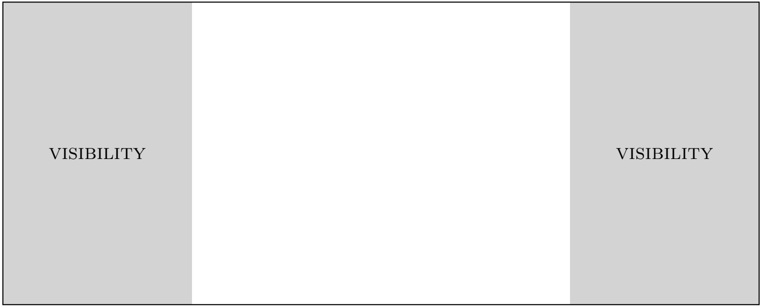
  - Lower bound on the time dedicated to higher priority experiment: $T_k(a, b)$
- Induced constraint:

$$Makespan(\Omega) \geq \frac{(\sum_{t_{ki} \in \Omega} \pi_i p_i) - M_k}{\tau} + T_k(a, b)$$

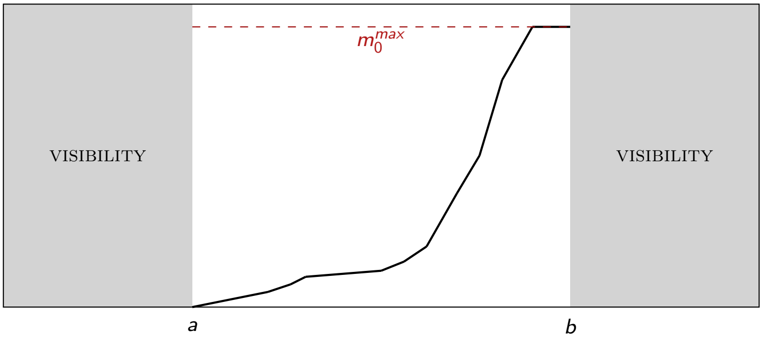
Propagation: visibility

- When the mass memory is full, no more data is transferred to it



Propagation: visibility

- When the mass memory is full, no more data is transferred to it
- Minimal usage and peak m_0^{max}



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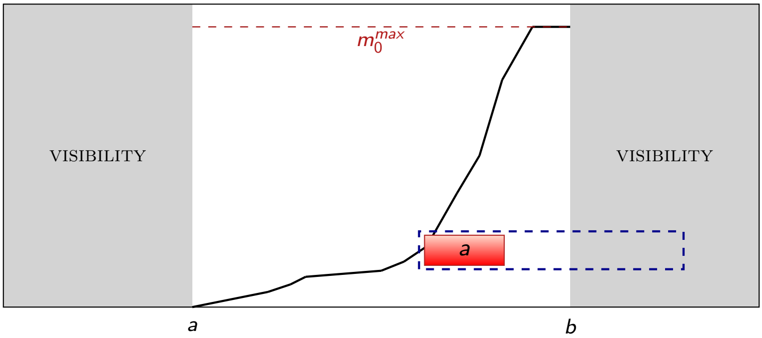

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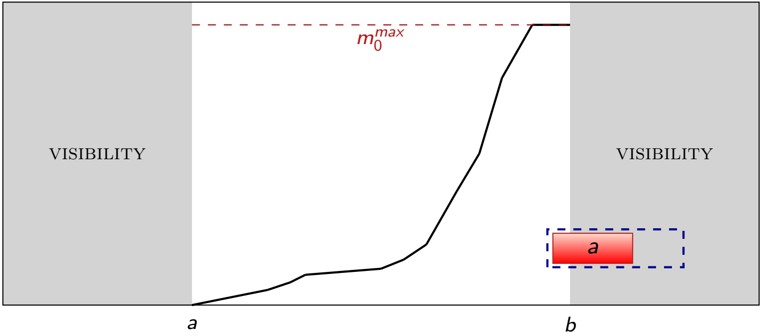
- Minimal usage and peak m_0^{max}
- If the production exceeds $M_k + M_0 - m_0^{max}$ in $[a, b]$ then data will be lost
  - Filtering: bound start time w.r.t. this quantity of data and production rate



Conclusions

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- Hybrid approaches (CP & MIP, CP & SAT,...)

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