Constraint Programming in a Nutshell

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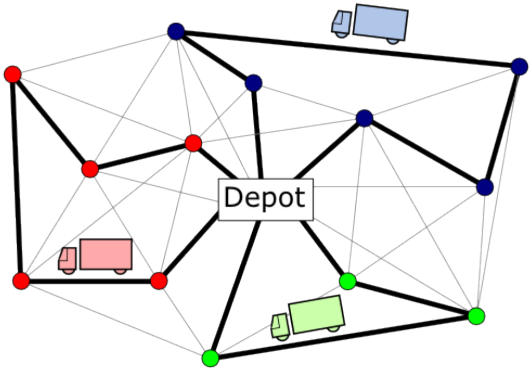
Optimisation



Optimisation is a science of service:
to scientists, to engineers, to artists, and to society.

Outline

Outline

Example (Agricultural experiment design)

	plot1	plot2	plot3	plot4	plot5	plot6	plot7
barley							
corn							
millet							
oats							
rye							
spelt							
wheat							

Constraints to be **satisfied**:

1. Equal sample size: Every grain is grown in 3 plots.
2. Equal growth load: Every plot grows 3 grains.
3. Balance: Every grain pair is grown in 1 common plot.

Instance: 7 grains, 7 plots, 3 plots/grain, 3 grains/plot, balance 1.

Example (Agricultural experiment design)

	plot1	plot2	plot3	plot4	plot5	plot6	plot7
barley	✓	✓	✓	–	–	–	–
corn	✓	–	–	✓	✓	–	–
millet	✓	–	–	–	–	✓	✓
oats	–	✓	–	✓	–	✓	–
rye	–	✓	–	–	✓	–	✓
spelt	–	–	✓	✓	–	–	✓
wheat	–	–	✓	–	✓	✓	–

Constraints to be **satisfied**:

1. Equal sample size: Every grain is grown in 3 plots.
2. Equal growth load: Every plot grows 3 grains.
3. Balance: Every grain pair is grown in 1 common plot.

Instance: 7 grains, 7 plots, 3 plots/grain, 3 grains/plot, balance 1.

Example (Doctor rostering)

	Mon	Tue	Wed	Thu	Fri	Sat	Sun
Doctor A							
Doctor B							
Doctor C							
Doctor D							
Doctor E							

Constraints to be **satisfied**:

1. #doctors-on-call / day $= 1$
2. #operations / workday ≤ 2
3. #operations / week ≥ 7
4. #appointments / week ≥ 4
5. day off after operation day
6. ...

Objective function to be **minimised**:

- Cost: ...

Example (Doctor rostering)

	Mon	Tue	Wed	Thu	Fri	Sat	Sun
Doctor A	call	–	oper	–	oper	–	–
Doctor B	app	call	–	oper	–	–	call
Doctor C	oper	–	call	app	app	call	–
Doctor D	app	oper	–	call	oper	–	–
Doctor E	oper	–	oper	–	call	–	–

Constraints to be **satisfied**:

1. #doctors-on-call / day $= 1$
2. #operations / workday ≤ 2
3. #operations / week ≥ 7
4. #appointments / week ≥ 4
5. day off after operation day
6. ...

Objective function to be **minimised**:

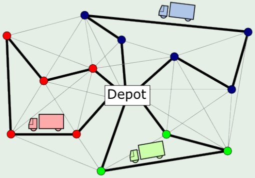
- Cost: ...

Example (Vehicle routing: parcel delivery)

Given a depot with a vehicle fleet and parcels for clients, **find** which vehicle brings which parcel to which client when.

Constraints to be **satisfied**:

1. All parcels are delivered on time.
2. No vehicle is overloaded.
3. Driver regulations are respected.
4. …



Objective function to be **minimised**:

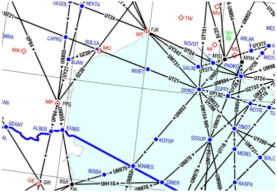
- Cost: the total fuel consumption and driver salary.

Example (Travelling salesperson: optimisation TSP)

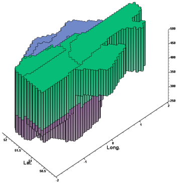
Given a map and cities, **find** a **shortest** route visiting each city once and returning to the starting city.

Applications in Air Traffic Management

Demand vs capacity



Airspace sectorisation



Contingency planning

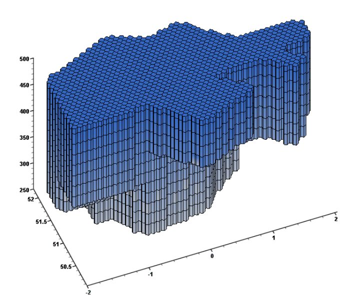
Flow	Time Span	Hourly Rate
From: Arlanda To: west, south	00:00 – 09:00 09:00 – 18:00 18:00 – 24:00	3 5 2
From: Arlanda To: east, north	00:00 – 12:00 12:00 – 24:00	4 3
...	...	...

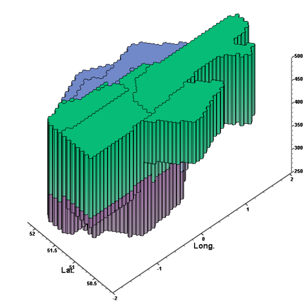
Workload balancing

Example (Airspace sectorisation)

Given an airspace split into c cells, and a targeted number s of sectors.

Find a colouring of the cells into s connected convex sectors, with minimal imbalance of the workloads of their air traffic controllers.

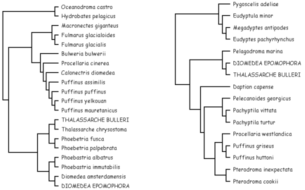




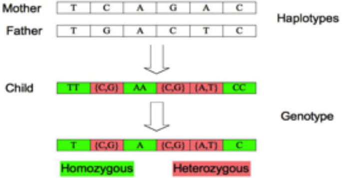
There are s^c possible colourings, but very few optimally satisfy the constraints: is intelligent search necessary?

Applications in Biology and Medicine

Phylogenetic supertree



Haplotype inference



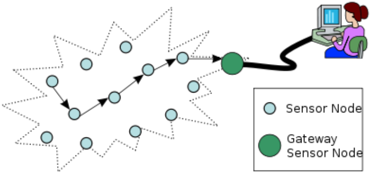
Medical image analysis

Doctor rostering

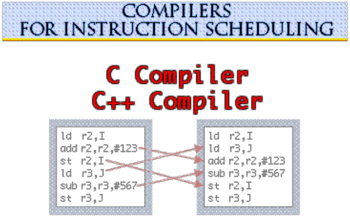
Applications in Programming and Testing

Robot-task sequencing

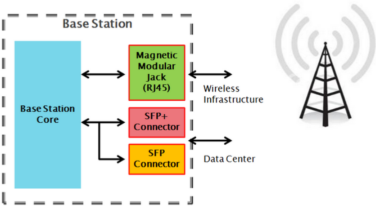
Sensor-net configuration



Compiler design

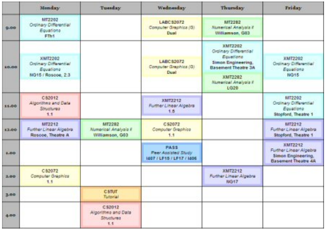


Base-station testing



Other Application Areas

School timetabling



	Monday	Tuesday	Wednesday	Thursday	Friday
9.00	MT2202 *Ordinary Differential Equations* FTh1		LABCS2072 *Computer Graphics (G)* Dual	MT2282 *Numerical Analysis II* Williamson, G03	
10.00	XMT2202 *Ordinary Differential Equations* NG15 / Roscoe, 2.3		LABCS2072 *Computer Graphics (G)* Dual	XMT2202 *Ordinary Differential Equations* Simon Engineering, Basement Theatre 3A XMT2282 *Numerical Analysis II* LG20	XMT2202 *Ordinary Differential Equations* NG15
11.00	CS2012 *Algorithms and Data Structures* 1.1		XMT2212 *Further Linear Algebra* 1.5		MT2202 *Ordinary Differential Equations* Stopford, Theatre 1
12.00	MT2212 *Further Linear Algebra* Roscoe, Theatre A	MT2282 *Numerical Analysis II* Williamson, G03	CS2072 *Computer Graphics* 1.1		MT2212 *Further Linear Algebra* Stopford, Theatre 1
1.00			PASS *Peer Assisted Study* I407 / LF15 / LF17 / I406		XMT2212 *Further Linear Algebra* Simon Engineering, Basement Theatre 4A
2.00	CS2072 *Computer Graphics* 1.1			XMT2212 *Further Linear Algebra* NG17	
3.00		CSTUT *Tutorial*			
4.00		CS2012 *Algorithms and Data Structures* 1.1			

Sports tournament design

Security: SQL injection?

Container packing

Definition

In a constraint problem, values have to be **found** for all the unknowns, called variables (in the mathematical sense) and ranging over **given** sets called domains, so that:

- All the given constraints on the variables are **satisfied**.
- Optionally: A given objective function on the variables has an optimal value: **minimal** cost or **maximal** profit.

Definition

A candidate solution to a constraint problem assigns to each variable a value within its domain.
The search space consists of all candidate solutions.

Example (Optimisation TSP over n cities)

A brute-force algorithm evaluates all $n!$ candidate routes:

- A computer of today evaluates 10^6 routes / second:

n	time
11	40 seconds
14	1 day
18	203 years
20	77k years

- Planck time is shortest useful interval: $\approx 5.4 \cdot 10^{-44}$ s; a Planck computer would evaluate $1.8 \cdot 10^{43}$ routes / s:

n	time
37	0.7 seconds
41	20 days
48	$1.5 \cdot$ age of universe

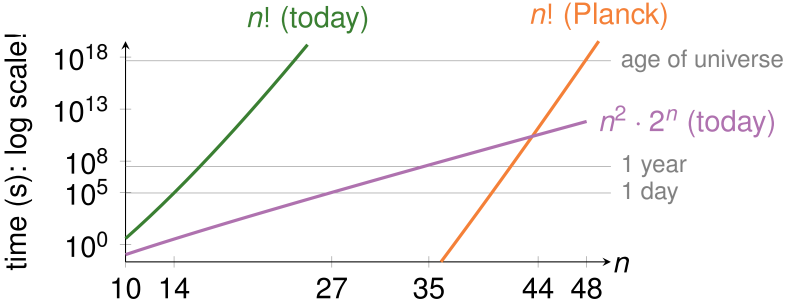
The dynamic program by Bellman-Held-Karp “only” takes $\mathcal{O}(n^2 \cdot 2^n)$ time: a computer of today takes a day for $n = 27$, a year for $n = 35$, the age of the universe for $n = 67$, and it beats the $\mathcal{O}(n!)$ algo on the Planck computer for $n \geq 44$.

Search spaces are often larger than the universe!

Many important real-life problems are NP-hard and can only be solved exactly & fast enough by intelligent search, unless P $=$ NP:

NP-hardness is not where the fun ends, but where it begins!

Do not give up but try to stay ahead of the curve: there is an instance size until which an **exact** algorithm is fast enough!



The Concorde TSP Solver beats the Bellman-Held-Karp exact algo: it uses approximation & local-search algorithms, but it can sometimes prove the exactness (optimality) of its solutions. The largest instance it has solved exactly, in 136 CPU years in 2006, has 85,900 cities! ☞ Let the fun begin!

Outline

A solving technology offers methods and tools for:

what: **Modelling** constraint problems in declarative language.

and / or

how: **Solving** constraint problems intelligently:

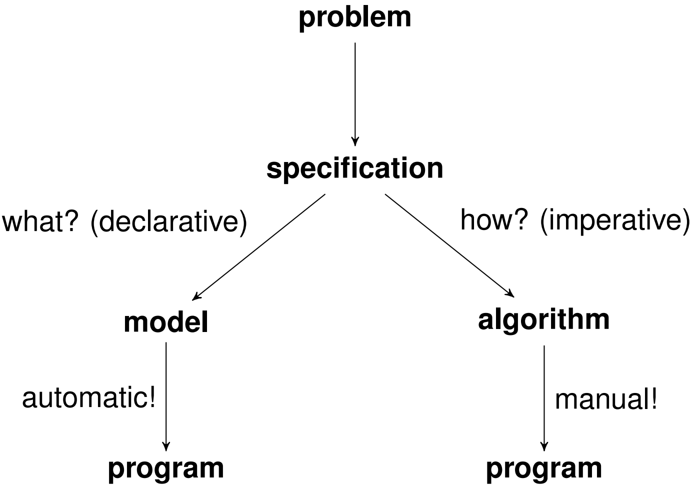
- Search: Explore the space of candidate solutions.
- Inference: Reduce the space of candidate solutions.
- Relaxation: Exploit solutions to easier problems.

A solver is a software that takes a model as input and tries to solve the modelled problem.

Combinatorial (= discrete) optimisation covers satisfaction *and* optimisation problems, for variables over *discrete* sets.

The ideas in this lecture extend to continuous optimisation, to soft optimisation, and to stochastic optimisation.

Modelling vs Programming



Example (Solving technologies)

With general-purpose solvers, taking a model as input:

- Boolean satisfiability (SAT)
- SAT modulo theories (SMT)
- (Mixed) integer linear programming (IP and MIP)
- Constraint programming (CP)
- ...
- Hybrid technologies

Techniques, *usually without* modelling and solvers:

- Dynamic programming (DP)
- Greedy algorithms
- Approximation algorithms
- Stochastic local search (SLS)
- Genetic algorithms (GA)
- ...

Constraint Programming Technology

Constraint programming (CP) offers methods & tools for:

what: Modelling constraint problems in a high-level language.

and

how: **Solving** constraint problems intelligently by:

- either default search upon pushing a button
- or systematic search guided by user-given strategies
- or local search guided by user-given (meta-)heuristics
- or hybrid search

plus inference, called propagation, but little relaxation.

Slogan of CP:

Constraint Program = Model [+ Search]

Outline

Example (Agricultural experiment design, AED)

	plot1	plot2	plot3	plot4	plot5	plot6	plot7
barley	✓	✓	✓	–	–	–	–
corn	✓	–	–	✓	✓	–	–
millet	✓	–	–	–	–	✓	✓
oats	–	✓	–	✓	–	✓	–
rye	–	✓	–	–	✓	–	✓
spelt	–	–	✓	✓	–	–	✓
wheat	–	–	✓	–	✓	✓	–

Constraints to be **satisfied**:

1. Equal sample size: Every grain is grown in 3 plots.
2. Equal growth load: Every plot grows 3 grains.
3. Balance: Every grain pair is grown in 1 common plot.

Instance: 7 grains, 7 plots, 3 plots/grain, 3 grains/plot, balance 1.
General term: balanced incomplete block design (BIBD).

Example (Agricultural experiment design, AED)

	plot1	plot2	plot3	plot4	plot5	plot6	plot7
barley	1	1	1	0	0	0	0
corn	1	0	0	1	1	0	0
millet	1	0	0	0	0	1	1
oats	0	1	0	1	0	1	0
rye	0	1	0	0	1	0	1
spelt	0	0	1	1	0	0	1
wheat	0	0	1	0	1	1	0

Constraints to be **satisfied**:

1. Equal sample size: Every grain is grown in 3 plots.
2. Equal growth load: Every plot grows 3 grains.
3. Balance: Every grain pair is grown in 1 common plot.

Instance: 7 grains, 7 plots, 3 plots/grain, 3 grains/plot, balance 1.
General term: balanced incomplete block design (BIBD).

Example (BIBD *integer* model: $\checkmark \rightsquigarrow 1$ and $- \rightsquigarrow 0$)

```
int: nbrVarieties; int: nbrBlocks;
set of int: Varieties = 1..nbrVarieties;
set of int: Blocks = 1..nbrBlocks;
int: sampleSize; int: blockSize; int: balance;
array[Varieties,Blocks] of var 0..1: BIBD;
solve satisfy;
constraint forall(v in Varieties)
 (sampleSize = sum(BIBD[v,..]));
constraint forall(b in Blocks)
 (blockSize = sum(BIBD[..,b]));
constraint forall(v1, v2 in Varieties where v1 < v2)
 (balance = sum(b in Blocks)(BIBD[v1,b]*BIBD[v2,b]));
```

Example (Instance data for our AED)

```
nbrVarieties = 7; nbrBlocks = 7;
sampleSize = 3; blockSize = 3; balance = 1;
```

Reconsider the model fragment:

```
constraint forall(v1, v2 in Varieties where v1 < v2)
 (balance = sum(b in Blocks)(BIBD[v1,b]*BIBD[v2,b]));
```

This constraint is declarative (and by the way non-linear), so read it using only the verb “to be” or synonyms thereof:

> *For all two ordered varieties* `v1` *and* `v2`*,*
> *the sum over all blocks* `b` *of the products*
> `BIBD[v1,b]*BIBD[v2,b]` *must equal* `balance`

The constraint is not procedural:

> *For all two ordered varieties* `v1` *and* `v2`*,*
> *we first add up, over all blocks* `b`*, the products*
> `BIBD[v1,b]*BIBD[v2,b]`*, and then we check*
> *whether that sum is equal to* `balance`

The latter reading is appropriate for solution checking, but solution finding performs no such procedural summation.

Example (Idea for another BIBD model)

barley	{plot1, plot2, plot3}
corn	{plot1, plot4, plot5}
millet	{plot1, plot6, plot7}
oats	{plot2, plot4, plot6}
rye	{plot2, plot5, plot7}
spelt	{plot3, plot4, plot7}
wheat	{plot3, plot5, plot6}

Constraints to be **satisfied**:

1. Equal sample size: Every grain is grown in 3 plots.
2. Equal growth load: Every plot grows 3 grains.
3. Balance: Every grain pair is grown in 1 common plot.

Example (BIBD *set* model: a block *set* per variety)

```
...
...
...
...
array[Varieties] of var set of Blocks: BIBD;
...
...
  (sampleSize = card(BIBD[v]));
...
  (blockSize = count(BIBD,b));
...
  (balance = card(BIBD[v1] inter BIBD[v2]));
```

Example (Instance data for our AED)

```
...
...
```

Example (Doctor rostering)

	Mon	Tue	Wed	Thu	Fri	Sat	Sun
Doctor A	call	–	oper	–	oper	–	–
Doctor B	app	call	–	oper	–	–	call
Doctor C	oper	–	call	app	app	call	–
Doctor D	app	oper	–	call	oper	–	–
Doctor E	oper	–	oper	–	call	–	–

Constraints to be **satisfied**:

1. #doctors-on-call / day $= 1$
2. #operations / workday ≤ 2
3. #operations / week ≥ 7
4. #appointments / week ≥ 4
5. day off after operation day
6. ...

Objective function to be **minimised**:

- Cost: ...

Example (Doctor rostering)

```
set of int: Days = 1..7;
set of int: Mon2Fri = 1..5;
enum: Doctors = {Dr A, Dr B, Dr C, Dr D, Dr E};
enum: ShiftTypes = {app, call, oper, none};

array[Doctors,Days] of var ShiftTypes: Roster;

solve minimize ...;  % objective function

constraint forall(d in Days)
  (count(Roster[..,d],call) = 1);
constraint forall(w in Mon2Fri)
  (count(Roster[..,w],oper) <= 2);
constraint count(Roster,oper) >= 7;
constraint count(Roster,app) >= 4;
constraint forall(d in Doctors)
  (regular(Roster[d,..], (oper none|app|call|none)*);
...  % other constraints
```

	6		1		4		5	
		8	3		5	6		
2								1
8			4		7			6
		6				3		
7			9		1			4
5								2
		7	2		6	9		
	4		5		8		7	

Example (Sudoku)

```
array[1..9,1..9] of var 1..9: Sudoku;
...  % load the hints
solve satisfy;
constraint forall(row in 1..9)
    (alldifferent(Sudoku[row,..]));
constraint forall(col in 1..9)
    (alldifferent(Sudoku[..,col]));
constraint forall(i,j in {1,4,7})
    (alldifferent(Sudoku[i..i+2,j..j+2]));
```

Using variables as indices: black magic?!

Example (Job allocation at minimal salary cost)

Given jobs `1..n` and the salaries of workers `1..w`,
find a worker for each job,
such that some constraints (on the qualifications of the workers for the jobs, on workload distribution, etc) are satisfied and the total salary cost is minimal:

```
array[1..w] of int: Salary;
array[1..n] of var 1..w: Worker; % job j by Worker[j]
solve minimize sum(j in 1..n)(Salary[Worker[j]]);
constraint ...;         % qualifications, workload, etc
```

Example (Travelling salesperson over cities `1..n`)

```
array[1..n,1..n] of float: Distance;  % instance data
array[1..n] of var 1..n: Next; % go from c to Next[c]
solve minimize sum(c in 1..n)(Distance[c,Next[c]]);
constraint circuit(Next);
constraint ...;                  % side constraints, if any
```

Constraint-Based Modelling

The given models are expressed in a high-level constraint-based modelling language, typical of CP:

- There are several **types** for variables: integers (`int`), reals (`float`), Booleans (`bool`), strings (`string`), integer sets (`set`), and matrices thereof (`array`).
- There is a nice vocabulary of **predicates** (<, <=, =, !=, >=, >, `alldifferent`, `circuit`, `regular`, ...), **functions** (+, −, *, `card`, `count`, `inter`, `sum`, ...), and **connectives** (/\, \/, ...).
- There is support for *both* constraint **satisfaction** (`satisfy`) *and* constrained **optimisation** (`minimize` and `maximize`).

Most modelling languages are (much) lower-level than this!

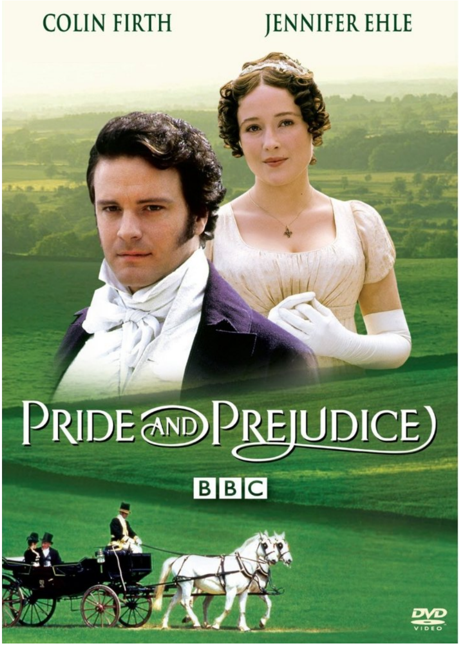
Got the Moves, But Can’t Show It?!

The constraint predicates (`alldifferent`, `circuit`, `regular`, ...) and structured variable types (sets, ...) allow us both to model the structure of a constraint problem and to exploit that structure when solving it.

Dozens of constraint predicates (see the Catalogue) declaratively encapsulate complex propagation algorithms, including for $\sum_{i=1}^{n} a_i \cdot x_i \sim b$, where $\sim \in \{<, \leq, =, \neq, \geq, >\}$.

If the scope of a predicate is an unfixed number of variables (an array of variables, a set variable, or a string variable), then one speaks of a global constraint in a CP model.

There is no standardised CP modelling language: distinct CP solvers may support distinct predicates, possibly under distinct names and signatures, as well as distinct types.
☞ But see the MiniZinc.org language & toolchain, which extends the spirit of AMPL and GAMS.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Pride:

Constraint programming represents one of the closest approaches computer science has yet made to the Holy Grail of programming: the user states the problem, the computer solves it.

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Prejudice:

The contribution of the article should be the reduction of an engineering problem to a known optimization format. [. . .] showcases pseudo code [. . .] submit this work to a journal interested in code semantics [. . .].
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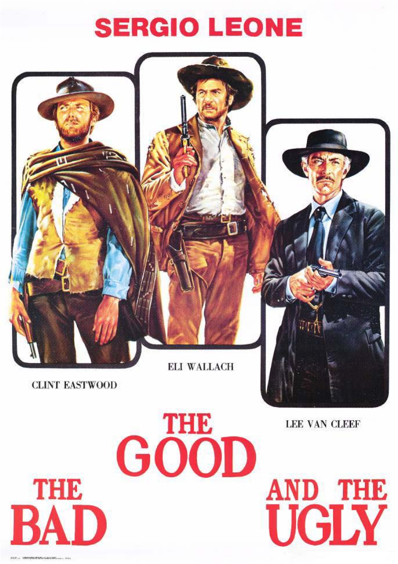
Correctness Is Not Enough for Models



Modelling is an Art!

There are good & bad models for each constraint problem:

- Different models of a problem may take different time on the same solver for the same instance.
- Different models of a problem may scale differently on the same solver for instances of growing size.

A tiny model change may accelerate the solving manyfold!

Good modellers are worth their weight in gold!

Use solvers, based on decades of cutting-edge research: you reuse hundreds of thousands of lines of highly tuned code that is very hard to beat on exact solving.

Outline

CP Technology (reminder)

Constraint programming (CP) offers methods & tools for:

what: Modelling constraint problems in a high-level language.

and

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plus inference, called propagation, but little relaxation.

Slogan of CP:

Constraint Program = Model [+ Search]

Outline

CP Solving = Propagation + Search

A CP solver conducts search interleaved with propagation:

Each constraint has a propagator.

Propagation of one Constraint: Propagator

Example

Consider the constraint CONNECTED($[C_1, \ldots, C_n]$), which enforces max one stretch per colour among the n variables.

From

. . . ? . . .			?	?	?			. . . ? . . .

the propagator of the CONNECTED(. . .) constraint infers

no redyellow								no redblack

☞ Propagation is the elimination of ~~the~~ impossible values from the current domains of the variables, and thereby accelerates otherwise blind search.

Example

Consider the n-ary predicate `alldifferent`, with $n = 4$:

$$\texttt{alldifferent([}a, b, c, d\texttt{])} \tag{1}$$

Modelling: (1) is equivalent to $\frac{n(n-1)}{2}$ binary constraints:

$$a \neq b \wedge a \neq c \wedge a \neq d \wedge b \neq c \wedge b \neq d \wedge c \neq d \tag{2}$$

Inference: (1) propagates much better than (2). Example:

$$a \in \{4, 5\},\ \ b \in \{4, 5\},\ \ c \in \{3, \cancel{4}\},\ \ d \in \{1, 2, \cancel{3}, \cancel{4}, \cancel{5}\}$$

No propagation by (2).

Example

Consider the n-ary predicate `alldifferent`, with $n = 4$:

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Modelling: (1) is equivalent to $\frac{n(n-1)}{2}$ binary constraints:

$$a \neq b \land a \neq c \land a \neq d \land b \neq c \land b \neq d \land c \neq d \tag{2}$$

Inference: (1) propagates much better than (2). Example:

$$a \in \{4, 5\},\ \ b \in \{4, 5\},\ \ c \in \{3, \cancel{4}\},\ \ d \in \{1, 2, \cancel{3}, \cancel{4}, \cancel{5}\}$$

No propagation by (2).

Example

Consider the n-ary predicate `alldifferent`, with $n = 4$:

$$\texttt{alldifferent([}a, b, c, d\texttt{])} \tag{1}$$

Modelling: (1) is equivalent to $\frac{n(n-1)}{2}$ binary constraints:

$$a \neq b \land a \neq c \land a \neq d \land b \neq c \land b \neq d \land c \neq d \tag{2}$$

Inference: (1) propagates much better than (2). Example:

$$a \in \{4, 5\},\ \ b \in \{4, 5\},\ \ c \in \{3, 4\},\ \ d \in \{1, 2, 3, 4, 5\}$$

No propagation by (2).

Example

Consider the n-ary predicate `alldifferent`, with $n = 4$:

$$\texttt{alldifferent}([a, b, c, d]) \tag{1}$$

Modelling: (1) is equivalent to $\frac{n(n-1)}{2}$ binary constraints:

$$a \neq b \wedge a \neq c \wedge a \neq d \wedge b \neq c \wedge b \neq d \wedge c \neq d \tag{2}$$

Inference: (1) propagates much better than (2). Example:

$$a \in \{4, 5\},\ \ b \in \{4, 5\},\ \ c \in \{3, \cancel{4}\},\ \ d \in \{1, 2, \cancel{3}, \cancel{4}, \cancel{5}\}$$

No propagation by (2). But perfect propagation by (1)!

Example

Consider the n-ary predicate `alldifferent`, with $n = 4$:

$$\texttt{alldifferent([}a, b, c, d\texttt{])} \tag{1}$$

Modelling: (1) is equivalent to $\frac{n(n-1)}{2}$ binary constraints:

$$a \neq b \wedge a \neq c \wedge a \neq d \wedge b \neq c \wedge b \neq d \wedge c \neq d \tag{2}$$

Inference: (1) propagates much better than (2). Example:

$$a \in \{4, 5\},\ b \in \{4, 5\},\ c \in \{3, \cancel{4}\},\ d \in \{1, 2, \cancel{3}, \cancel{4}, \cancel{5}\}$$

No propagation by (2). But perfect propagation by (1)!
Search: The `alldifferent` propagator is suspended, as its constraint currently does not surely hold.

Example

Consider the n-ary predicate `alldifferent`, with $n = 4$:

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Modelling: (1) is equivalent to $\frac{n(n-1)}{2}$ binary constraints:

$$a \neq b \wedge a \neq c \wedge a \neq d \wedge b \neq c \wedge b \neq d \wedge c \neq d \tag{2}$$

Inference: (1) propagates much better than (2). Example:

$$a \in \{4, 5\},\; b \in \{4, 5\},\; c \in \{3, \cancel{4}\},\; d \in \{1, 2, \cancel{3}, \cancel{4}, \cancel{5}\}$$

No propagation by (2). But perfect propagation by (1)!
Search: The `alldifferent` propagator is suspended, as its constraint currently does not surely hold. If search or another propagator infers $a = 4$, then the `alldifferent` propagator is awakened: it infers $b = 5$ and is disposed of, as its constraint then surely holds, for any d.

Example (Propagator for `alldifferent`)

Solutions to `alldifferent`$([u, v, w, x, y, z])$ map to maximum matchings in a bipartite graph for the domains:

$u \in \{0, 1\}$

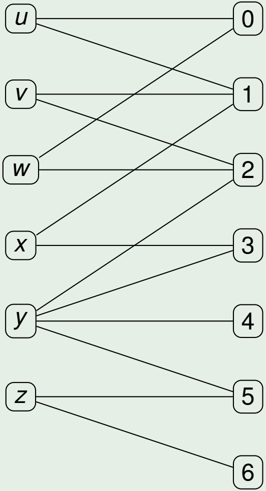
$v \in \{1, 2\}$

$w \in \{0, 2\}$

$x \in \{1, 3\}$

$y \in \{2, 3, 4, 5\}$

$z \in \{5, 6\}$



Example (Propagator for `alldifferent`)

Mark all edges of *some* maximum matching; Hopcroft-Karp algorithm takes $O(m\sqrt{n})$ time for n variables and m values:

$u \in \{0, 1\}$

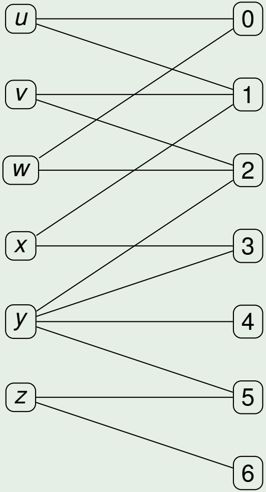
$v \in \{1, 2\}$

$w \in \{0, 2\}$

$x \in \{1, 3\}$

$y \in \{2, 3, 4, 5\}$

$z \in \{5, 6\}$



Example (Propagator for `alldifferent`)

Mark all edges of *some* maximum matching; Hopcroft-Karp algorithm takes $O(m\sqrt{n})$ time for n variables and m values:

$u \in \{0, 1\}$

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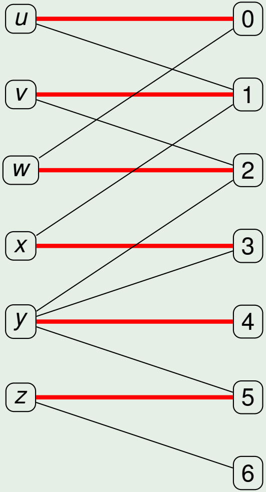
$w \in \{0, 2\}$

$x \in \{1, 3\}$

$y \in \{2, 3, 4, 5\}$

$z \in \{5, 6\}$



Example (Propagator for `alldifferent`)

Mark all other edges in *all* other maximum matchings, exploiting a result by J. Petersen in *Acta Mathematica* 1891:

$u \in \{0, 1\}$

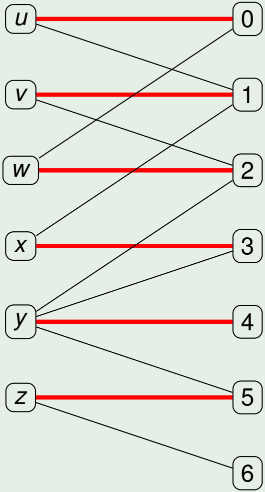
$v \in \{1, 2\}$

$w \in \{0, 2\}$

$x \in \{1, 3\}$

$y \in \{2, 3, 4, 5\}$

$z \in \{5, 6\}$



Example (Propagator for `alldifferent`)

Mark all other edges in *all* other maximum matchings, exploiting a result by J. Petersen in *Acta Mathematica* 1891:

$u \in \{0, 1\}$

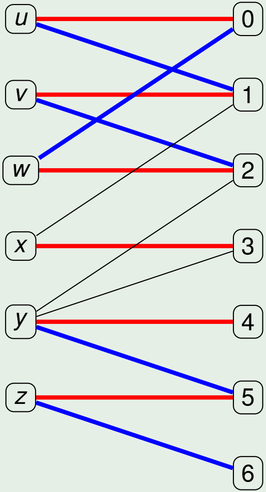
$v \in \{1, 2\}$

$w \in \{0, 2\}$

$x \in \{1, 3\}$

$y \in \{2, 3, 4, 5\}$

$z \in \{5, 6\}$



Example (Propagator for `alldifferent`)

Every still unmarked edge is in *no* maximum matching.
Propagate accordingly within the current domains:

$u \in \{0, 1\}$

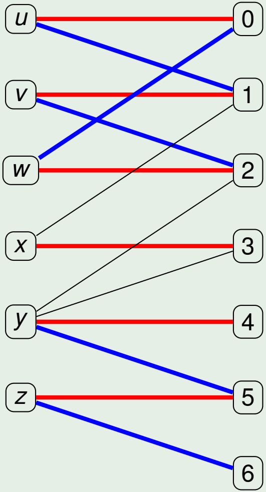
$v \in \{1, 2\}$

$w \in \{0, 2\}$

$x \in \{1, 3\}$

$y \in \{2, 3, 4, 5\}$

$z \in \{5, 6\}$



Example (Propagator for `alldifferent`)

Every still unmarked edge is in *no* maximum matching.
Propagate accordingly within the current domains:

$u \in \{0, 1\}$

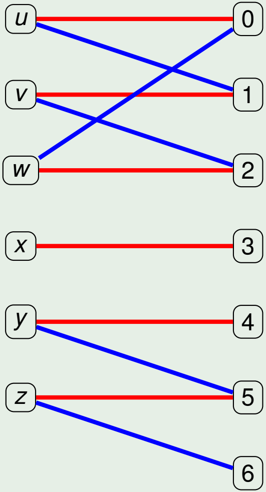
$v \in \{1, 2\}$

$w \in \{0, 2\}$

$x \in \{\cancel{1}, 3\}$

$y \in \{\cancel{2}, \cancel{3}, 4, 5\}$

$z \in \{5, 6\}$



Search + Propagation of All Constraints

Example (BIBD: AED partial assignment)

	plot1	plot2	plot3	plot4	plot5	plot6	plot7
barley	✓	✓	✓	–	–	–	–
corn	✓	–	–	✓	✓	–	–
millet	✓	–	–	–	–	✓	✓
oats	–	✓	–	✓	–	✓	–
rye	?						
spelt							
wheat							

Search + Propagation of All Constraints

Example (BIBD: AED partial assignment)

	plot1	plot2	plot3	plot4	plot5	plot6	plot7
barley	✓	✓	✓	–	–	–	–
corn	✓	–	–	✓	✓	–	–
millet	✓	–	–	–	–	✓	✓
oats	–	✓	–	✓	–	✓	–
rye	?						
spelt							
wheat							

But plot1 cannot grow rye as that would violate the second constraint (every plot grows 3 grains).

Search + Propagation of All Constraints

Example (BIBD: AED partial assignment)

	plot1	plot2	plot3	plot4	plot5	plot6	plot7
barley	✓	✓	✓	–	–	–	–
corn	✓	–	–	✓	✓	–	–
millet	✓	–	–	–	–	✓	✓
oats	–	✓	–	✓	–	✓	–
rye	–						
spelt							
wheat							

But plot1 cannot grow rye as that would violate the second constraint (every plot grows 3 grains).

Search + Propagation of All Constraints

Example (BIBD: AED partial assignment)

	plot1	plot2	plot3	plot4	plot5	plot6	plot7
barley	✓	✓	✓	–	–	–	–
corn	✓	–	–	✓	✓	–	–
millet	✓	–	–	–	–	✓	✓
oats	–	✓	–	✓	–	✓	–
rye	–						
spelt							
wheat							

But plot1 cannot grow rye as that would violate the second constraint (every plot grows 3 grains). Actually, plot1 cannot grow oats, spelt, or wheat either, for the same reason, and this was already propagated when trying the search guess that plot1 grow millet!

Search + Propagation of All Constraints

Example (BIBD: AED partial assignment)

	plot1	plot2	plot3	plot4	plot5	plot6	plot7
barley	✓	✓	✓	–	–	–	–
corn	✓	–	–	✓	✓	–	–
millet	✓	–	–	–	–	✓	✓
oats	–	✓	–	✓	–	✓	–
rye	–						
spelt	–						
wheat	–						

But plot1 cannot grow rye as that would violate the second constraint (every plot grows 3 grains). Actually, plot1 cannot grow oats, spelt, or wheat either, for the same reason, and this was already propagated when trying the search guess that plot1 grow millet!

Example (BIBD: AED partial assignment)

	plot1	plot2	plot3	plot4	plot5	plot6	plot7
barley	✓	✓	✓	–	–	–	–
corn	✓	–	–	✓	✓	–	–
millet	✓	–	–	–	–	✓	✓
oats	–	✓	–	✓	–	✓	–
rye	–	?					
spelt	–						
wheat	–						

Example (BIBD: AED partial assignment)

	plot1	plot2	plot3	plot4	plot5	plot6	plot7
barley	✓	✓	✓	–	–	–	–
corn	✓	–	–	✓	✓	–	–
millet	✓	–	–	–	–	✓	✓
oats	–	✓	–	✓	–	✓	–
rye	–	?					
spelt	–						
wheat	–						

Guess: Let plot2 grow rye. (Strategy: ✓ guesses first.)

Example (BIBD: AED partial assignment)

	plot1	plot2	plot3	plot4	plot5	plot6	plot7
barley	✓	✓	✓	–	–	–	–
corn	✓	–	–	✓	✓	–	–
millet	✓	–	–	–	–	✓	✓
oats	–	✓	–	✓	–	✓	–
rye	–	✓					
spelt	–						
wheat	–						

Guess: Let plot2 grow rye. (Strategy: ✓ guesses first.)

Example (BIBD: AED partial assignment)

	plot1	plot2	plot3	plot4	plot5	plot6	plot7
barley	✓	✓	✓	–	–	–	–
corn	✓	–	–	✓	✓	–	–
millet	✓	–	–	–	–	✓	✓
oats	–	✓	–	✓	–	✓	–
rye	–	✓					
spelt	–						
wheat	–						

Propagation: plot2 cannot grow spelt and wheat as otherwise the second constraint (every plot grows 3 grains) would be violated for plot2.

Example (BIBD: AED partial assignment)

	plot1	plot2	plot3	plot4	plot5	plot6	plot7
barley	✓	✓	✓	–	–	–	–
corn	✓	–	–	✓	✓	–	–
millet	✓	–	–	–	–	✓	✓
oats	–	✓	–	✓	–	✓	–
rye	–	✓					
spelt	–	–					
wheat	–	–					

Propagation: plot2 cannot grow spelt and wheat as otherwise the second constraint (every plot grows 3 grains) would be violated for plot2.

Example (BIBD: AED partial assignment)

	plot1	plot2	plot3	plot4	plot5	plot6	plot7
barley	✓	✓	✓	–	–	–	–
corn	✓	–	–	✓	✓	–	–
millet	✓	–	–	–	–	✓	✓
oats	–	✓	–	✓	–	✓	–
rye	–	✓					
spelt	–	–					
wheat	–	–					

Propagation: plot3, plot4, and plot6 cannot grow rye as otherwise the third constraint (every grain pair is grown in 1 common plot) would be violated.

Example (BIBD: AED partial assignment)

	plot1	plot2	plot3	plot4	plot5	plot6	plot7
barley	✓	✓	✓	–	–	–	–
corn	✓	–	–	✓	✓	–	–
millet	✓	–	–	–	–	✓	✓
oats	–	✓	–	✓	–	✓	–
rye	–	✓	–	–		–	
spelt	–	–					
wheat	–	–					

Propagation: plot3, plot4, and plot6 cannot grow rye as otherwise the third constraint (every grain pair is grown in 1 common plot) would be violated.

Example (BIBD: AED partial assignment)

	plot1	plot2	plot3	plot4	plot5	plot6	plot7
barley	✓	✓	✓	–	–	–	–
corn	✓	–	–	✓	✓	–	–
millet	✓	–	–	–	–	✓	✓
oats	–	✓	–	✓	–	✓	–
rye	–	✓	–	–		–	
spelt	–	–					
wheat	–	–					

Propagation: plot5 and plot7 must grow rye as otherwise the first constraint (every grain is grown in 3 plots) would be violated for rye.

Example (BIBD: AED partial assignment)

	plot1	plot2	plot3	plot4	plot5	plot6	plot7
barley	✓	✓	✓	–	–	–	–
corn	✓	–	–	✓	✓	–	–
millet	✓	–	–	–	–	✓	✓
oats	–	✓	–	✓	–	✓	–
rye	–	✓	–	–	✓	–	✓
spelt	–	–					
wheat	–	–					

Propagation: plot5 and plot7 must grow rye as otherwise the first constraint (every grain is grown in 3 plots) would be violated for rye.

Example (BIBD: AED partial assignment)

	plot1	plot2	plot3	plot4	plot5	plot6	plot7
barley	✓	✓	✓	–	–	–	–
corn	✓	–	–	✓	✓	–	–
millet	✓	–	–	–	–	✓	✓
oats	–	✓	–	✓	–	✓	–
rye	–	✓	–	–	✓	–	✓
spelt	–	–					
wheat	–	–					

Propagation: plot3 must grow spelt and wheat as otherwise the second constraint (every plot grows 3 grains) would be violated for plot3.

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Example (BIBD: AED partial assignment)

	plot1	plot2	plot3	plot4	plot5	plot6	plot7
barley	✓	✓	✓	–	–	–	–
corn	✓	–	–	✓	✓	–	–
millet	✓	–	–	–	–	✓	✓
oats	–	✓	–	✓	–	✓	–
rye	–	✓	–	–	✓	–	✓
spelt	–	–	✓				
wheat	–	–	✓				

Propagation: plot3 must grow spelt and wheat as otherwise the second constraint (every plot grows 3 grains) would be violated for plot3.

Example (BIBD: AED partial assignment)

	plot1	plot2	plot3	plot4	plot5	plot6	plot7
barley	✓	✓	✓	–	–	–	–
corn	✓	–	–	✓	✓	–	–
millet	✓	–	–	–	–	✓	✓
oats	–	✓	–	✓	–	✓	–
rye	–	✓	–	–	✓	–	✓
spelt	–	–	✓				
wheat	–	–	✓				

Common fixpoint reached: No more propagation possible.

Example (BIBD: AED partial assignment)

	plot1	plot2	plot3	plot4	plot5	plot6	plot7
barley	✓	✓	✓	–	–	–	–
corn	✓	–	–	✓	✓	–	–
millet	✓	–	–	–	–	✓	✓
oats	–	✓	–	✓	–	✓	–
rye	–	✓	–	–	✓	–	✓
spelt	–	–	✓	✓			
wheat	–	–	✓				

Guess: Let plot4 grow spelt. (Strategy: ✓ guesses first.)

Propagation: etc.

Systematic search, for a satisfaction problem:

1: propagate all constraints; **backtrack** if empty domain
2: **if** only fixed variables, **then** show solution & **backtrack**
3: **while** there is at least one suspended propagator **do**
4: select unfixed variable, v, of current domain $\mathrm{dom}(v)$
5: partition $\mathrm{dom}(v)$ using guesses (say $v = d$ & $v \neq d$, or $v > d$ & $v \leq d$, for a selected value $d \in \mathrm{dom}(v)$)
6: **for each** guess: **recurse** upon adding it as constraint

For an optimisation problem: before backtracking at line 2 add the constraint that any next solution must be better.

Strategies:

- Line 4: variable selection strategy: smallest domain, ...
- Line 5: value selection strategy: maximum, median, ...
- Line 5: guess selection strategy: equality, bisection, ...
- Tree exploration: depth-first search, ...

Example (Search for the BIBD *integer* model)

```
solve :: int_search(BIBD,input_order,indomain_max) satisfy;
```

Outline

Example (BIBD: AED assignment after *i* moves)

	plot1	plot2	plot3	plot4	plot5	plot6	plot7
barley	✓	✓	✓	–	–	–	–
corn	✓	–	–	✓	–	✓	–
millet	✓	–	–	–	–	✓	✓
oats	–	✓	–	✓	✓	–	–
rye	–	✓	–	–	✓	–	✓
spelt	–	–	✓	✓	–	–	✓
wheat	–	–	✓	–	✓	✓	–

1. Equal sample size: Every grain is grown in 3 plots. Satisfied at initialisation and by each move: invariant.
2. Equal growth load: Every plot grows 3 grains. Currently satisfied: zero violation.
3. Balance: Every grain pair is grown in 1 common plot. But, e.g., oats & rye are grown in $2 > 1$ common plots.

Example (BIBD: AED assignment after *i* moves)

	plot1	plot2	plot3	plot4	plot5	plot6	plot7
barley	✓	✓	✓	–	–	–	–
corn	✓	–	–	✓	–	✓	–
millet	✓	–	–	–	–	✓	✓
oats	–	✓	–	✓	✓	–	–
rye	–	✓	–	–	✓	–	✓
spelt	–	–	✓	✓	–	–	✓
wheat	–	–	✓	–	✓	✓	–

1. Equal sample size: Every grain is grown in 3 plots. Satisfied at initialisation and by each move: invariant.
2. Equal growth load: Every plot grows 3 grains. Currently satisfied: zero violation.
3. Balance: Every grain pair is grown in 1 common plot. But, e.g., oats & rye are grown in $2 > 1$ common plots.

Selected move: let plot6 instead of plot5 grow oats.

Example (BIBD: AED assignment after *i* moves)

	plot1	plot2	plot3	plot4	plot5	plot6	plot7
barley	✓	✓	✓	–	–	–	–
corn	✓	–	–	✓	–	✓	–
millet	✓	–	–	–	–	✓	✓
oats	–	✓	–	✓	–	✓	–
rye	–	✓	–	–	✓	–	✓
spelt	–	–	✓	✓	–	–	✓
wheat	–	–	✓	–	✓	✓	–

1. Equal sample size: Every grain is grown in 3 plots. Satisfied at initialisation and by each move: invariant.
2. Equal growth load: Every plot grows 3 grains. Currently satisfied: zero violation.
3. Balance: Every grain pair is grown in 1 common plot. But, e.g., oats & rye are grown in $2 > 1$ common plots.

Selected move: let plot6 instead of plot5 grow oats.

Example (BIBD: AED assignment after $i + 1$ moves)

	plot1	plot2	plot3	plot4	plot5	plot6	plot7
barley	✓	✓	✓	–	–	–	–
corn	✓	–	–	✓	–	✓	–
millet	✓	–	–	–	–	✓	✓
oats	–	✓	–	✓	–	✓	–
rye	–	✓	–	–	✓	–	✓
spelt	–	–	✓	✓	–	–	✓
wheat	–	–	✓	–	✓	✓	–

1. Equal sample size: Every grain is grown in 3 plots. Satisfied at initialisation and by each move: invariant.
2. Equal growth load: Every plot grows 3 grains. But plot5 grows $2 < 3$ grains; plot6 grows $4 > 3$ grains.
3. Balance: Every grain pair is grown in 1 common plot. But, e.g., corn & oats are grown in $2 > 1$ common plots.

Example (BIBD: AED assignment after $i+1$ moves)

	plot1	plot2	plot3	plot4	plot5	plot6	plot7
barley	✓	✓	✓	–	–	–	–
corn	✓	–	–	✓	–	✓	–
millet	✓	–	–	–	–	✓	✓
oats	–	✓	–	✓	–	✓	–
rye	–	✓	–	–	✓	–	✓
spelt	–	–	✓	✓	–	–	✓
wheat	–	–	✓	–	✓	✓	–

1. Equal sample size: Every grain is grown in 3 plots. Satisfied at initialisation and by each move: invariant.
2. Equal growth load: Every plot grows 3 grains. But plot5 grows $2 < 3$ grains; plot6 grows $4 > 3$ grains.
3. Balance: Every grain pair is grown in 1 common plot. But, e.g., corn & oats are grown in $2 > 1$ common plots.

Selected move: let plot5 instead of plot6 grow corn.

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Example (BIBD: AED assignment after $i + 1$ moves)

	plot1	plot2	plot3	plot4	plot5	plot6	plot7
barley	✓	✓	✓	–	–	–	–
corn	✓	–	–	✓	✓	–	–
millet	✓	–	–	–	–	✓	✓
oats	–	✓	–	✓	–	✓	–
rye	–	✓	–	–	✓	–	✓
spelt	–	–	✓	✓	–	–	✓
wheat	–	–	✓	–	✓	✓	–

1. Equal sample size: Every grain is grown in 3 plots. Satisfied at initialisation and by each move: invariant.
2. Equal growth load: Every plot grows 3 grains. But plot5 grows $2 < 3$ grains; plot6 grows $4 > 3$ grains.
3. Balance: Every grain pair is grown in 1 common plot. But, e.g., corn & oats are grown in $2 > 1$ common plots.

Selected move: let plot5 instead of plot6 grow corn.

Example (BIBD: AED assignment after $i + 2$ moves)

	plot1	plot2	plot3	plot4	plot5	plot6	plot7
barley	✓	✓	✓	–	–	–	–
corn	✓	–	–	✓	✓	–	–
millet	✓	–	–	–	–	✓	✓
oats	–	✓	–	✓	–	✓	–
rye	–	✓	–	–	✓	–	✓
spelt	–	–	✓	✓	–	–	✓
wheat	–	–	✓	–	✓	✓	–

1. Equal sample size: Every grain is grown in 3 plots. Satisfied at initialisation and by each move: invariant.
2. Equal growth load: Every plot grows 3 grains. Currently satisfied: zero violation.
3. Balance: Every grain pair is grown in 1 common plot. Currently satisfied: zero violation.

Stop search: All constraints are satisfied.

Local search:

1: let s and s^* be the same computed initial assignment
2: **while** there are violated constraints & iterations left **do**
3:     select a move on s; let s' be the reached assignment
4:     **if** s' is better than s^* **then** $s^* := s'$
5:     $s := s'$
6: **return** s^*

Heuristics: What move to select?

- Line 3: assign, flip, swap, add, drop, transfer, ...
- Line 3: best / first / random improvement, ...

Meta-heuristics: How to escape local optima?

- Lines 2 to 5: simulated annealing, tabu search, ...

Heuristics are an Art!

There are good & bad heuristics for each problem model:

- Different heuristics for a model may take different time on the same solver for the same instance.
- Different heuristics for a model may scale differently on the same solver for instances of growing size.

A tiny heuristic tweak may accelerate the solving manyfold!

Good heuristicians are worth their weight in gold!

Use solvers, based on decades of cutting-edge research: you reuse hundreds of thousands of lines of highly tuned code that is very hard to beat.

Outline

Stand-alone languages:

- ALICE by Jean-Louis Laurière, France, 1976
- CHIP at ECRC, Germany, 1987 – 1990,
  then marketed by Cosytec, France
- OPL, by P. Van Hentenryck, USA, and ILOG, France: front-end to both ILOG CP Optimizer and ILOG CPLEX
- Comet, by P. Van Hentenryck and L. Michel, USA
- MiniZinc, at U. of Melbourne and Monash U., Australia
- ...

Libraries (the ones listed before ";" are open-source):

- Prolog: ECLiPSe, . . . ; SICStus Prolog, . . .
- C++: Gecode, OR-Tools; IBM CP Optimizer, CHIP, . . .
- Java: Choco, Google OR-Tools, JaCoP, . . . ; . . .
- Scala: OscaR; . . .
- ...

Outline

Success Stories by CP Users and Contributors:

AIMMS

AMPL

BOUYGUES

cadence

CISCO

FICO

Google

IBM

intel

JEPPESEN
A BOEING COMPANY

NASA

ORACLE

QUINTIQ

RedPrairie

RENAULT

SAP

SIEMENS

TACTON

THALES

XEROX

. . .

Success stories: CP = **technology of choice** in scheduling, configuration, personnel rostering, timetabling, . . .

Outline

Scope of Constraint Programming

CP has a wide scope, as it addresses:

- satisfaction problems and optimisation problems
- discrete variables and continuous variables
- linear constraints and non-linear constraints

in principle in any combinations thereof, by:

- systematic search, if optimality more crucial than speed
- local search, if speed is more crucial than optimality

Common Misconceptions about CP

- CP is for experts: the predicate vocabulary is *large*.

  But there are tools helping you identify predicates, and about a dozen predicates take you a very long way!

- CP is for experts: the search strategy is *mandatory*.

  No, it is optional: there is adaptive autonomous search!

- CP'ers claim CP is a silver bullet for NP-hard problems.

  No: CP solvers are complementary in strength to MIP, SAT, SMT, ... solvers and to local search, which leads to hybrid optimisation technologies: LCG, ... !

Bluffer's Guide to the C$\ast$ Alphabet Soup

- CP solvers $=$ constraint solvers

  MIP, SAT, ... solvers are also constraint solvers, whether known as such in those communities or not!

- CP $=$ CLP $=$ constraint *logic* programming

  Many modern CP solvers are not Prolog libraries!

  $$\text{CLP}(\mathbb{Q}, \mathbb{R}) \text{ solvers} \subsetneq \text{CLP solvers} \subsetneq \text{CP solvers}$$

  CP is a granddaughter of logic programming (LP)!

- CP $=$ CSP $=$ constraint satisfaction *programming*

  CSP $=$ constraint satisfaction *problem*, an AI term.

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Opportunities for CP

Rapid prototyping, with high solving performance, when:

- Constraints are, still or again, subject to experiments
- Partition into hard & soft constraints yet undetermined

Combinatorial structure is impure, due to side constraints.

It is time to consider all or more problem constraints.

Domain knowledge exploitable for problem-specific search.

It is a configuration problem.

It is a personnel rostering problem.

It is a scheduling problem.

It is a time-tabling problem.

Outline

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